EXAM 1 **TQS 120** Fall 2010

NAME: Key

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions, and x, y, and z be non-zero real numbers.

T (F) $\frac{f(x)}{\frac{1}{2}} = f(x)\frac{1}{2}$ $\frac{f(x)}{\frac{1}{2}} = 2f(x)$

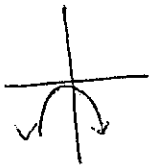
T (F) $(x+y)^2 = x^2 + y^2$ $(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

T (F) $3x^{-2} = \frac{1}{3x^2}$ $3x^{-2} = \frac{3}{x^2}$

T (F) $3x^{-2} = 3x^{\frac{1}{2}}$ $3x^{\frac{1}{2}} = 3\sqrt{x}$

(T) F $(3+5i) - (4-2i) = -1 + 7i$ ✓

(T) F The graph of $y = -x^2$ is increasing from $(-\infty, 0)$.



T (F) -2 is a root of $y = 3(2x+2)(x-1)(x+4)$ $3(-4+2)(-3)(2) \neq 0$

(T) F $\frac{3+5i}{1-2i} = \frac{-7}{5} + \frac{11}{5}i$ ✓ ✓ $\frac{3+5i}{1-2i}\cdot\frac{(1+2i)}{(1+2i)} = \frac{3+6i+5i-10}{1-4i^2}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] (Practice Exam1#2) Find all x so that:

$$3(7+x)^{-2} = 6$$

stated (+.5)
neg exp (+1)
sq'd (+.5)
otherwise (+.5)
order of op (+.5)

$\frac{3}{(7+x)^2} = 6$

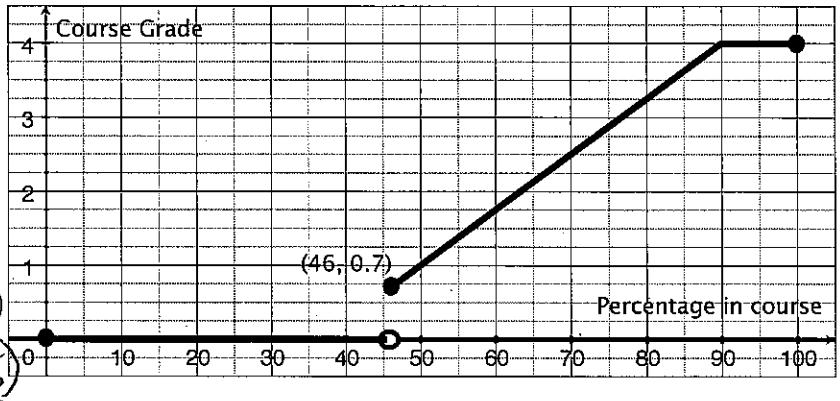
$3 = 6(7+x)^2$

$\frac{1}{2} = (7+x)^2$

$\pm\sqrt{\frac{1}{2}} = 7 + x$

$-7 \pm \sqrt{\frac{1}{2}} = x$

3. The graph of a piece-wise defined function G is provided on the right.



(a) [1] (§2.1 #26)
Estimate $G(60)$.

1.75

(b) [2] (§2.7 #27)
Estimate $(G \circ G)(80)$

$G(G(80)) = G(3.25)$ (+.5) (+1)

$= 0$ (+.5)

(c) [2] (§2.7 #11) Estimate $G(70) - G(80)$.

$\approx 2.5 - 3.25 = 4.05 - .75$ (+1) (+.5) (+.5)

(d) [2] (Quiz 1#1b) What is the range of G?

$\{0\} \cup [.7, 4.0]$ (+.5) (+.5) (+.5)

end points (+.5)

(e) [3] (Quiz 1#1c) Find a formula for the function G in the indicated form.

$$G(x) = \begin{cases} 0 & \text{if } 0 \le x < 46 \\ .075x - 2.75 & \text{if } 46 \le x < 90 \\ 4.0 & \text{if } 90 \le x \le 100 \end{cases}$$

(+1) (+1)

line between 46 & 90

pts on line: (46, .7) (90, 4.0) (+.5)

$\frac{4.0 - .7}{90 - 46} = \frac{3.3}{44}$

$y = mx + b$

$4.0 = \frac{3.3}{44} \cdot 90 + b$

$4.0 - \frac{3.3 \cdot 90}{44} = b$

$y = \frac{3.3}{44}x + \left(4.0 - \frac{3.3 \cdot 90}{44}\right)$ (+.5)

4. Let $f(x) = \dfrac{1}{x+1}$.

(a) [2] (§2.1 #41) What is the domain of f?

$x+1 \neq 0$ (+1)

$x \neq -1$ (+1)

or $(-\infty, -1) \cup (-1, \infty)$

(b) [3] (§2.8 #19) The function f is one-to-one. Find $f^{-1}(5)$.

$x = \dfrac{1}{y+1}$ (+1)

$x(y+1) = 1$ (dom +.5)

$xy + x = 1$

$xy = 1 - x$

$y = \dfrac{1-x}{x}$

alg +1.5

(c) [4] (§2.8 #37) The function f is one-to-one. Find $f^{-1}(x)$.

$f^{-1}(x) = \dfrac{1-x}{x}$

$\therefore f^{-1}(5) = \dfrac{1-5}{5} = -\dfrac{4}{5}$

or

f^{-1}: $\star \leftarrow 5$ stated (+.5)

what input $\star$ so that output 5 (+.5)

$f(\star) = 5$

$\dfrac{1}{\star+1} = 5$

$1 = 5(\star+1)$

$1 = 5\star + 5$

$-4 = 5\star$

$\star = -\dfrac{4}{5}$

alg (+1)

(d) [5] (Quiz 1#2c) Find and simplify the *difference quotient* of f at 3, that is find and simplify

$$\frac{f(3+h) - f(3)}{h}.$$

$$\frac{f(3+h)-f(3)}{h} = \frac{\frac{1}{3+h+1} - \frac{1}{3+1}}{h} = \frac{\frac{1}{4+h} - \frac{1}{4}}{h} = \frac{\frac{4}{4(4+h)} - \frac{4+h}{4(4+h)}}{h}$$

(+1) (+1) (+1) Same denominator (+1)

$$= \frac{4-(4+h)}{4(4+h)} \div \frac{h}{1} = \frac{-h}{4(4+h)} \cdot \frac{1}{h} = \frac{-1}{4(4+h)} \checkmark$$

alg (+.5) sign (+.5)

5. [4] (§1.2 #63) Simplify the given expression

$$\frac{(2x^4y^{\frac{-4}{5}})^3(8y^2)^{\frac{2}{3}}}{}$$

stated (+.5) dist exponent (+1) power to power (+1) simplify (+.5) coef (+1)

$$= 8x^{12}2^{2}y^{-12/5}y^{4/3} = 8\cdot 4x^{12}y^{-12/5+4/3} = 32x^{12}y^{-16/15}$$

$-\frac{12}{5}+\frac{4}{3}$

$\frac{-36+20}{15}$

$\frac{-16}{15}$

6. Let $m(x) = 2x^3 + 2x^2 - 28x + 12$ and $n(x) = 2x^2 + 8x - 4$.

(a) [3] (§2.5 #15) Complete the square to write n in vertex (standard) form.

$y = 2x^2+8x-4$

$\frac{1}{2}y = x^2+4x-2$

$+4 \qquad +\left(\frac{4}{2}\right)^2$

$\frac{1}{2}y+4 = x^2+4x+4-2$

$\frac{1}{2}y+4 = (x+2)^2-2$

$\frac{1}{2}y = (x+2)^2-6$

$y = 2(x+2)^2-12$

by leading coeff (+1) add $(\frac{1}{2})^2$ (+1) alg (+.5) form (+.5)

(b) [4] (§3.2 #17) Is n a factor of m? That is, does $2x^2 + 8x - 4$ divide into $2x^3 + 2x^2 - 28x + 12$ with no remainder? Justify your answer.

stated (+.5) know ? (+.5) / setup (+.5) alg (+1.5) interpret (+1)

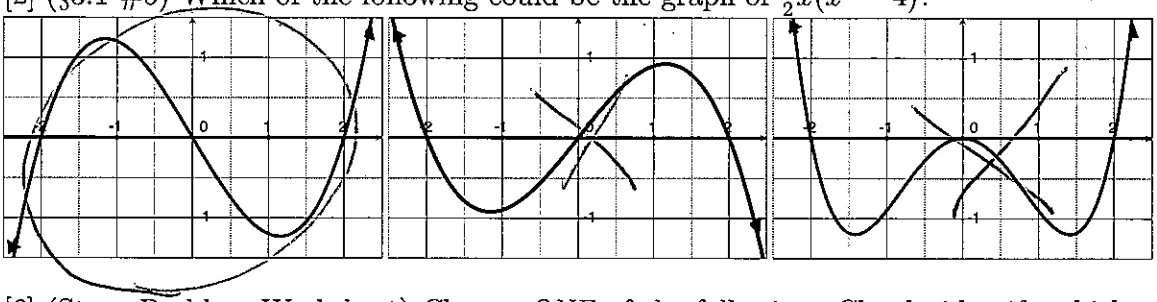
$$\begin{array}{r} x-3 \quad R0 \\ 2x^2+8x-4 \overline{)\,2x^3+2x^2-28x+12} \\ -(2x^3+8x^2-4x) \\ \hline -6x^2-24x+12 \\ -(-6x^2-24x+12) \\ \hline 0 \end{array}$$

yes, n is a factor b/c

$(2x^2+8x-4)(x-3) = 2x^3+2x^2-28x+12$

leading term is pos. cubic

7. [2] (§3.1 #5) Which of the following could be the graph of $\frac{1}{2}x(x^2-4)$? $(x+2)(x-2)$ roots @ 0 & ± 2



8. [6] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

   (a) (Word Problems Worksheet) You would like to set the price for a UWT fund-raising raffle. You did a similar thing last year and when you set the price to \$6 about 63 people bought tickets. The stats class did some research for you and reported that if ticket prices reduced by \$3.15, sales would increase by about 21 tickets. What price should you set the tickets so as to maximize income from ticket sales (to the nearest penny)?

   (b) (Word Problems Worksheet) Potassium ferrate has been considered for use in batteries but costs \$100 per gram. You have a battery case that is currently *full* with 50 grams of a mixture that is 10% potassium ferrate. You would like to build the battery but you need a higher concentration of the potassium ferrate (40% should do it). What is the minimum amount of potassium ferrate you will have to buy and add to the battery case (after you dumped out some of the original mixture to make room) to get the cathode to work?

started variables (+1)

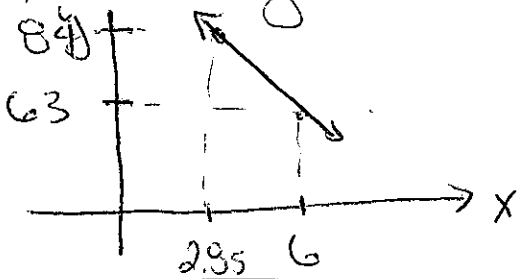
a) income = (price per ticket)(# tickets sold)
$= x \cdot y$ (+1)

want to maximize income.

Note the following relationship:

x	y
\$6	63
2.85	84

linear relation?

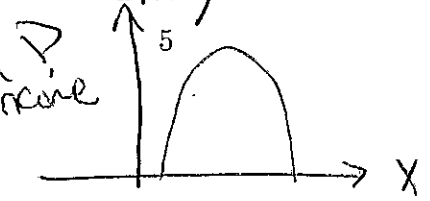


connection between x & y (+1)

$y = mx + b$
$y = \frac{21}{-3.15}x + b$
$63 = \frac{-21}{3.15} \cdot 6 + b$
$b = 63 + \frac{126}{3.15} = 103$

So
$\text{income} = x \cdot y$
$= x\left(\frac{-21}{3.15}x + \left(63 + \frac{126}{3.15}\right)\right)$
sub (+1)

which is a parabola?
$= x(-6.6x + 103)$
$= -6.6x^2 + 103x$



we just need the vertex... (+5)

found vertex (+1) \$7.73
max is \$397.24

~~7~~ 15
~~10~~ 25
~~14~~ 10
~~4~~ 50
~~8~~

Have 50 grams of mixture of 5 grams is potassium ferrate
I want 50 gram mixture with 20 grams of potassium ferrate

Let x be the amount of potassium ferrate I add &
let y be the amount of ~~need~~ orginal mixture I keep.

potassium ferrate from original mix	+	Potassium ferrate I add	=	Potassium ferrate amount I want

$$.1y + x = 20 \quad (*)$$

(+1) (+1) (+.5)

Notice because the battery only has room for 50 grams

$$x + y = 50 \quad (+1)$$

$$\Rightarrow y = 50 - x$$

Sub this in to (*) and we have

$$.1(50-x) + x = 20$$

$$\Rightarrow 5 - .1x + x = 20$$

$$5 + .9x = 20$$

$$.9x = 15$$

$$x \approx 16.67 \text{ grams}$$

(+1.5)

So I'll need to dump out 16.7 grams of the original mixture & add 16.7 grams of potassium ferrate.

Notice I'll have to buy $\frac{\$100}{gr} \cdot 16.7gr = \$1670$ worth of potassium ferrate, which while expensive is better than dumping out the mixture & starting from nothing & buying $\frac{\$100}{gr} \cdot 20gr = \$2000$. That \$330 in savings?