Exam 1

TQS 120

Practice

Key

NAME: This is a sample exam to be used for practice. This is *not* a template for the exam that will be given in class. Many of the questions on the exam will look quite different than those appearing here.

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T F $\frac{3x+y}{3z} = \frac{x+y}{z}$ $\qquad \frac{x+y}{z} = \frac{3(x+y)}{3z} \neq \frac{3x+y}{3z}$

T F $(x+y)^2 = x^2 + y^2$ $\qquad (x+y)^2 = (x+y)(x+y) = x^2 + xy + yx + y^2$

T F $|x| = x$ $\qquad |-1| = 1 \neq -1$

T F $\frac{3+5i}{1-2i} = -\frac{7}{5} + \frac{11}{5}i$ $\qquad \frac{3+5i}{1-2i}\frac{(1+2i)}{(1+2i)} = \frac{3+11i-10}{1+4} = \frac{3}{5} - \frac{10}{5} + \frac{11}{5}i = -\frac{7}{5} + \frac{11}{5}i$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] Given $3(7+x)^{-2} - 4 = 2$, solve for x.

$$\frac{3}{(7+x)^2} - 4 = 2$$

$$\frac{3}{(7+x)^2} = 6$$

$$3 = 6(7+x)^2$$

$$\frac{1}{2} = (7+x)^2$$

$$\pm\sqrt{\tfrac{1}{2}} = 7 + x$$

$$-7 \pm \sqrt{\tfrac{1}{2}} = x$$

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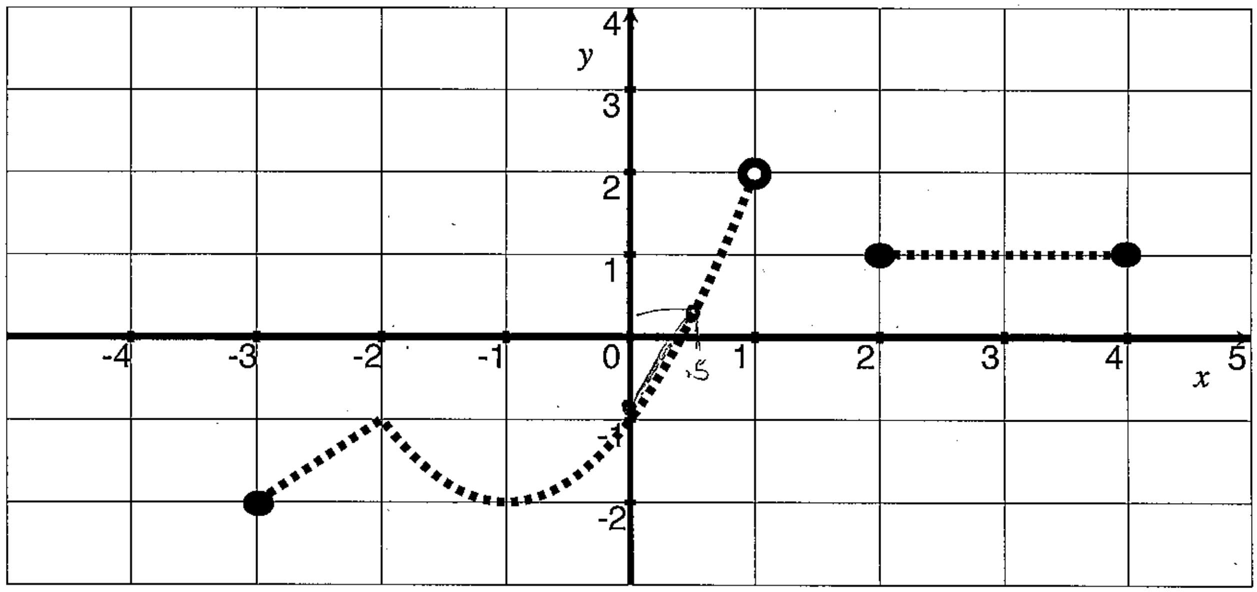
3. [4] Let f be the function whose graph is below:



Estimate the following *if* possible:

$f(-3)$

-2

$\frac{f(-3)-1}{f(-1)} = \frac{-2-1}{-2} = \frac{-3}{-2} = 3/2$

$f(1)$ does not exist

$(f \circ f)(-2) = f(f(-2)) = f(-1) = -2$

$f(-1)f(2) = (-2)\cdot 1 = -2$

$f(0+.5) = f(1/2) \approx .3$

$\frac{f(0+.5)}{.5} = \frac{.3}{.5} = 3/5$

The average rate of change of f from $x = 0$ to $x = .5$

Slope of line $\frac{\text{rise}}{\text{run}} = \frac{1.3}{.5} = \frac{13}{5}$

$\frac{f(.5)-f(0)}{.5}$

4. [4] Find the domain of g where $g(x) = \frac{2-\sqrt{5-2x}}{x+10}$.

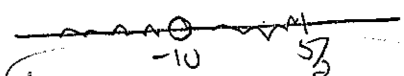
need

\# under sqroots ≥ 0 And den $\neq 0$

$5-2x \geq 0$ $\qquad$ $x+10 \neq 0$

$-2x \geq -5$ $\qquad$ $x \neq -10$

$x \leq 5/2$



so $x \leq 5/2$ and $x \neq -10$ or $(-\infty,-10) \cup (-10, 5/2]$

5. [4] Consider the points $P = (3, 4)$ and $Q = (-1, -2)$. Find the equation to a line that goes through the point $(1, 1)$ and has a perpendicular slope to the line connecting P and Q.

slope of $\overleftrightarrow{PQ} = \frac{4+2}{3+1} = \frac{6}{4} = \frac{3}{2}$

$\perp$ slope is $-2/3$

passes thru $(1,1)$ so

$1 = -2/3(1) + b$

$5/3 = 1 + 2/3 = b$ $\qquad \therefore \quad y = -2/3\, x + 5/3$

- [1] What is the y intercept of the line you found?

when $x=0$ so $y = -2/3 \cdot 0 + 5/3 = 5/3$

$(0, 5/3)$

- [1] Find the zeros of the line you found above.

zeros when $y=0$

so $0 = -2/3 \cdot x + 5/3$

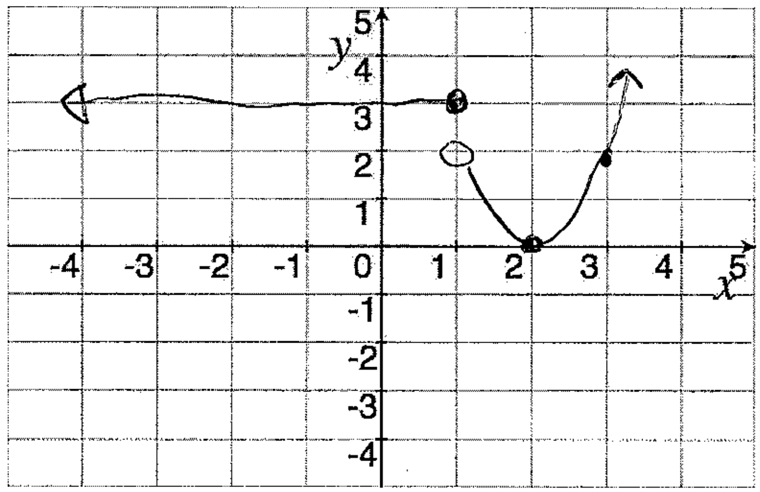
$-\frac{3}{2} \cdot \; -5/3 = \cancel{-2/3} \cdot x \cdot \cancel{-3/2}$

$5/2 = x$ so $(5/2, 0)$

check: $-2/3 \cdot 5/2 + 5/3 = -5/3 + 5/3 = 0$ ✓

6. Let h be the *function* defined by:

$$h(x) = \begin{cases} 3 & x \le 1 \\ 2(x-2)^2 & 1 < x \end{cases}$$



$2(x-2)^2$
vert stretch by 2
shift right by 2

(a) [3] Graph h. (Explaining graph transformations is worth partial credit.)

(b) [] What are the coordinates of the vertex on the piece of the graph above that is a parabola?

$(2,0)$

(c) [2] Find all possible input(s) so that $h(x) = 1$.

when does $1 = 2(x-2)^2 \Rightarrow \frac{1}{2} = (x-2)^2 \Rightarrow \pm\sqrt{\frac{1}{2}} = x-2$

$\Rightarrow x = 2 \pm \sqrt{\frac{1}{2}}$

(d) What is the range h?

$[0, \infty)$

7. [4] Given that $j(x) = -3x^2 + 6x - 2$. Write j in vertex (standard) form.

$\cancel{-3y+6}$

$-3x^2 + 6x - 2 = y$

$x^2 - 2x + 2/3 = -1/3\, y$

$+1 \qquad +1$

$(x^2 - 2 + 1) + 2/3 = -1/3\, y + 1$

$(x-1)^2 + 2/3 = -1/3\, y + 1$

$(x-1)^2 + 2/3 - 1 = -1/3\, y$

$(x-1)^2 - 1/3 = -1/3\, y$

$-3(x-1)^2 + 1 = y$

8. [4] Simplify the following as much as possible:

$$\frac{(2x^4y^{\frac{1}{6}})^3(6xy^3)^{-3}}{8^{\frac{-2}{3}}x^4y^4}$$

$$\frac{2^3(x^4)^3(y^{1/6})^3 6^{-3} x^{-3}(y^3)^{-3}}{(8^{1/3})^{-2} x^4 y^4} = \frac{8\cancel{x^{12}}\cancel{y^{1/2}} \frac{1}{6^3} \frac{1}{\cancel{x^3}} \frac{1}{\cancel{y^9}}}{2^{-2} x^4 y^4}$$

$$= \frac{8x^{12-3}y^{1/2-9}2^2}{6x^4y^4} = \frac{\cancel{2}\cdot2\cdot2\cdot2\cdot2\,x^9 y^{-17/2}}{\cancel{2}\cdot3 \quad x^4 y^4} = \frac{16}{3}\frac{x^{9-4}}{y^{4+17/2}}$$

$$= 16/3\, x^5 \frac{1}{y^{21/2}} = \frac{16}{3}x^5\sqrt{y^{21}}$$

9. [3] Find a cubic polynomial whose graph passes through the points $(-2,0)$ and $(1,0)$ and has a root at 6. Note: there are many correct answers possible here.

$-2, 1$ & 6 are roots

so $(x--2)(x-1)(x-6)$

$(x+2)(x-1)(x-6)$ works, so does $-4(x+2)(x-1)(x-6)$

10. Let $m(x) = x^3 + x^2 - \frac{39}{4}x + 9$ and $n(x) = x + 4$. Use long division to find $D(x)$ and $R(x)$ so that $\frac{m(x)}{n(x)} = D(x) + \frac{R(x)}{n(x)}$

$$\begin{array}{r} x^2 - 3x + 9/4 \quad R0 \\ x+4 \,\big)\, \overline{\cancel{x^3} + x^2 - 39/4\, x + 9} \\ -(\cancel{x^3} + 4x^2) \\ \hline \cancel{-3x^2} - 39/4\, x + 9 \\ -(\cancel{-3x^2} - 12x) \\ \hline 9/4\, x + 9 \\ -(9/4\, x + 9) \\ \hline 0 \end{array}$$

$-\frac{39}{4} + \frac{48}{4} = \frac{9}{4}$

$x^2 - 3x + 9/4 + \frac{0}{x+4}$

11. Let $p(x) = \dfrac{x-5}{7x+5} + 3$.

(a) Given that p is one-to-one (ie has an inverse), find p^{-1}.

$$x = \frac{y-5}{7y+5} + 3$$

$$x - 3 = \frac{y-5}{7y+5}$$

$$(7y+5)(x-3) = y-5$$

$$7xy - 21y + 5x - 15 = y - 5$$

$$7xy - 21y - y = 15 - 5x - 5$$

$$y(7x - 21 - 1) = 15 - 5x - 5$$

$$y = \frac{10-5x}{7x-22}$$

(b) Write the expression $p(a+h)$ and simplify.

$$p(a+h) = \frac{a+h-5}{7(a+h)+5} + 3 = \frac{a+h-5}{7a+7h+5} + 3$$

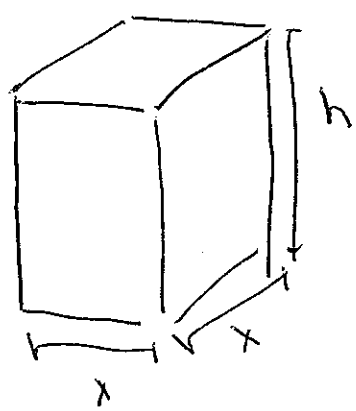
(c) Write the expression $\dfrac{p(a+h)-p(a)}{h}$ and simplify.

$$\frac{p(a+h)-p(a)}{h} = \left(\left[\frac{a+h-5}{7a+7h+5} + 3\right] - \left[\frac{a-5}{7a+5} + 3\right]\right) \div h = \left(\frac{a+h-5}{7a+7h+5} + \cancel{3} - \frac{a-5}{7a+5} - \cancel{3}\right) \div$$

$$= \frac{(a+h-5)(7a+5) - (a-5)(7a+7h+5)}{(7a+7h+5)(7a+5)} \cdot \frac{1}{h} = \frac{(\cancel{7a^2} + \cancel{5a} + \cancel{7ah} + 5h - \cancel{35a} - \cancel{25}) - (\cancel{7a^2} + \cancel{7ah} + \cancel{5a} - \cancel{35a} - 35h}{(7a+7h+5)(7a+5)h}$$

$$= \frac{5h - 35h}{(7a+7h+5)(7a+5)h} = \frac{-30\cancel{h}}{\cancel{h}(7a+7h+5)(7a+5)} = \frac{-30}{(7a+7h+5)(7a+5)}$$

12. A rectangular box with a volume of 60 ft^3 has a square base. Find a function that models its surface area S in terms of the length x of one side of its base.



$$x \cdot x \cdot h = 60\,ft^3 \Rightarrow h = \frac{60\,ft^3}{x^2}$$

$$S = xh + xh + xh + xh + x^2 + x^2 = 4xh + x^2$$

$$= 4x\left(\frac{60\,ft^3}{x^2}\right) + x^2 = \frac{240}{x} + x^2$$