Practice TQS 120 Final

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite di?erent than those appearing here.

[10] Let f & g, be functions.

T F $(f \circ g)(x) = (g \circ f)(x)$

T F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$

T F $\sqrt{(x^2)} = x$ for all real numbers x.

T F If 2 is a root of g, then $g(2) = 0$.

T F $\ln \frac{x}{y} = \ln x - \ln y$ for all positive numbers x and y.

T F $\log(\log(10)) = 0$.

T F $\sin^{-1}(\sin x) = x$ for all real numbers x.

T F $\sin(\frac{\pi}{3} + x) = \sin \frac{\pi}{3} + x$

T F If $\sin\theta > 0$ and $\tan\theta < 0$, then $\cos\theta < 0$

T F The range of $\sin^{-1}$ is $[0, \pi]$

Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. Find all x such that

$$2(5 + (8 - x)^2)^{-\frac{1}{2}} - 1 = 0$$

.

2. [2] Explain what a function is.

3. Given $m(x) = \dfrac{x}{x-5}$, and $n(x) = \sqrt{4x-8}$,

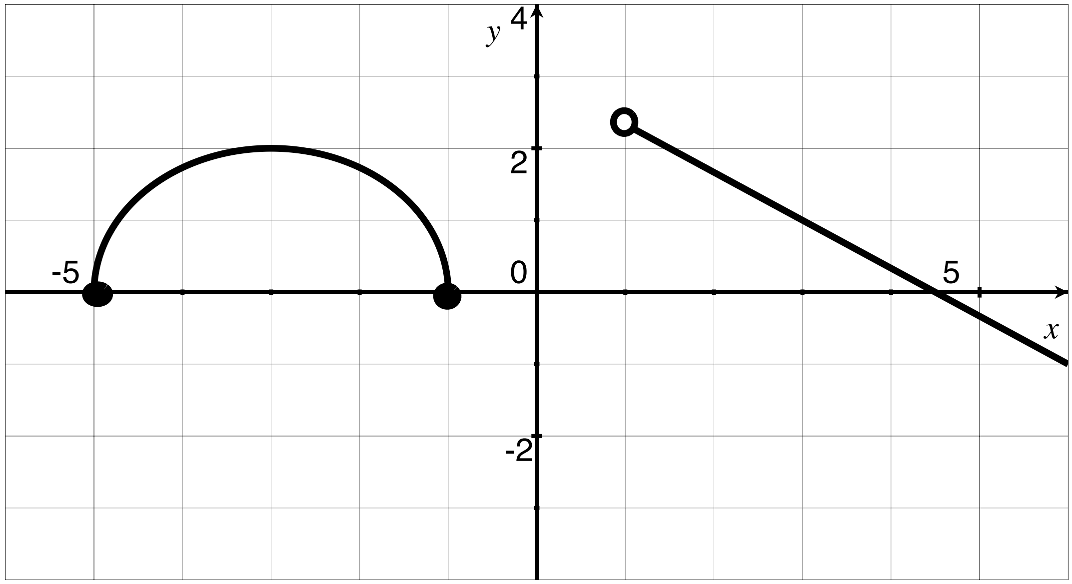
   (a) The function m passes the horizontal line test. Find m^{-1}.

   (b) [4] If $p(x) = 3m(x+1)$, find the domain and rule of p.

   (c) [3] Find the domain and rule of $n \circ m$.

   (d) [5] Find the domain and rule of $\dfrac{n}{m}$.

4. [3] Let the following be the graph of g.



(a) What is the domain of g?

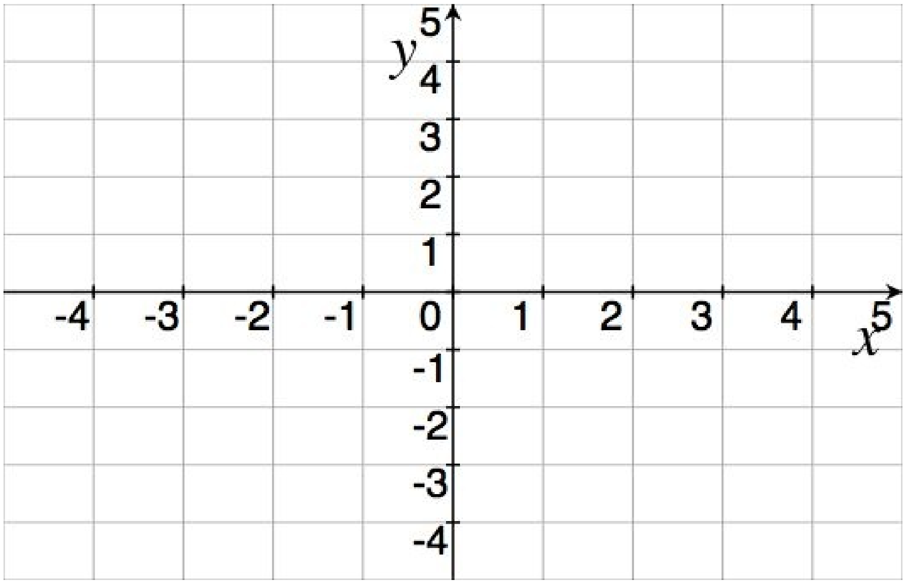
(b) The function g is a piecewise defined function consisting of a straight line and a semicircle. Write down the rule for g.

(c) Use the graph above to *estimate* all x value(s) so that $g(x) = 1$?

(d) Find the total length (of the curve and the line) that is graphed above.

5. Define f by

$$f(x) = \begin{cases} \tan x & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2 \\ \log_2(x-1) & \text{if } x \geq 2 \end{cases}$$



(a) [8] Graph f on the axes above.

(b) [9] Find the following if possible:

$f(1)$

$f(2) + f(3)$

$f(0)$

$f(\frac{-13\pi}{4})$

Range of f

6. [8] Find all of the exact values x that satisfy the following:

$2\sin x = -\sqrt{3}$

$5^{4x-1} = 7^x$

7. [4] Find all exact values for x that satisfy the following:

$\log(x-16) = 2 - \log(x-1)$

$3^{5x}9^x = 27$

8. Simplify:

$$\frac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}}$$

$2 - \log_5(25z)$

9. [7] Given $f(3) = 0$, use the factor theorem to find the other roots of $x^4 - 3x^3 - 25x^2 + 75x$

10. Simplify:

$\sin^{-1}(\sin \frac{3\pi}{4})$

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x}$$

11. [4] Let $\frac{-\pi}{2} < \theta < 0$ and $\cos\theta = \frac{1}{5}$. Find $\tan\theta$.

12. [6] Let $\frac{\pi}{2} < \phi < \pi$ and $\frac{-\pi}{2} < \theta < 0$. Given that $\sin\phi = \frac{2}{3}$ and that $\cos\theta = \frac{1}{5}$, find $\cos(\theta + \phi)$. (You are free to use results from #10 above.)

13. [5] Your given a 16 oz mocha that is a rather weak 3% espresso. You, knowing you'll be up late studying mathematics, would rather like a 30% espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso to have a 16 oz mocha with the desired concentration?

14. [5] Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope

15. [5] An aircraft tracking station determines the distance from a common point O to each aircraft and the angle between the aicrafts. If angle O between the two aircrafts is equal to $49°$ and the distances from point O to the two aircrafts are 50km and 72km, find distance d between the two aircrafts.