Exam 2 **TMath 115** Autumn 2020

NAME: Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T (F) $\frac{1}{b+1} + \frac{1}{b} = \frac{3}{b+1}$

$\frac{b}{b}\cdot\frac{1}{b+1} + \frac{1}{b}\cdot\frac{b+1}{b+1} = \frac{b+b+1}{b(b+1)} = \frac{2b+1}{b(b+1)}$

(T) F $-2(x+1)^2 - 7 = f(x)$ is a polynomial

$-2(x+1)^2-7 = -2(x^2+2x+1)-7 = -2x^2-4x-9$

T (F) $-2(x+1)^2 - 7 = f(x)$ has a root at $x = -1$

↳ vertex @ $(-1,-7)$ not a root/zero/x-intercept

(T) F $-2(x+1)^2(x-3)$ is a third degree polynomial.

$-2(x+1)(x+1)(x-3) = (-2x-2)(x^2-2x-3)$

$= -2x^3+4x^2+6x-2x^2+4x+6$

(T) F $i^3 = -i$

$i^3 = (i)i = (-1)\cdot i = -i$

Show all your work. Reasonable supporting work must be shown to earn credit.

2. [3] (WebHW6 #3) Compute: $(8+8i)(3-i)$

$24 - 8i + 24i - 8i^2$

$24 + 16i - 8(-1)$ (+.5)

$32 + 16i$

got it (+.5)

start (+.5)

FOIL (+.5)

real part (+.5)

im. part (+.5)

3. [3] (Exam1 #3) Perform and simplify $\frac{x-2}{\frac{x}{3}} + \frac{x}{5}$

order of op (+.5)

fraction in fraction (+1)

common den (+.5)

solution (+.5)

simplify (+.5)

$\frac{3}{3}\cdot\frac{x-2}{\frac{x}{3}} + \frac{x}{5}$

$\frac{5}{5}\cdot\frac{3x-6}{x} + \frac{x}{5}\cdot\frac{x}{x}$

$\frac{15x-30}{5x} + \frac{x^2}{5x}$

$\frac{x^2+15x-30}{5x}$

OR $(x-2) \div \frac{x}{3} = \frac{x-2}{1}\cdot\frac{3}{x} = \frac{3(x-2)}{x}$

$\frac{5}{5}\cdot\frac{3(x-2)}{x} + \frac{x}{5}\cdot\frac{x}{x}$

$\frac{15(x-2)}{5x} + \frac{x^2}{5x} = \frac{x^2+15x-30}{5x}$

4. [3] (§A.8 #52) Current (I), voltage (V), and impedance (Z) are often represented with complex numbers and relate to each other by $Z = \frac{V}{I}$. Given that $V = 35 + 70i$ and $I = 7 + 5i$, find Z.

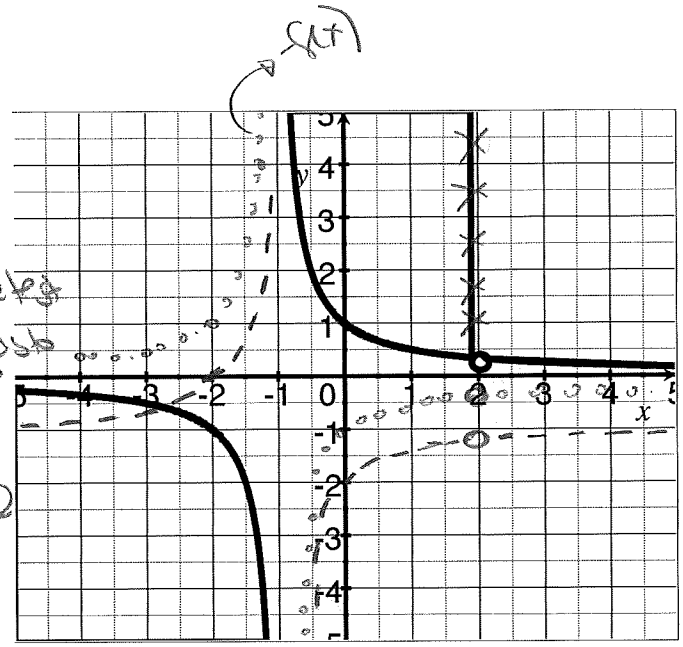
FOIL (+1)

$Z = \frac{35+70i}{7+5i}\cdot\frac{(7-5i)}{(7-5i)} = \frac{245-175i+490i-350i^2}{49-\cancel{35i}+\cancel{35i}-25i^2} = \frac{245+315i-350(-1)}{49-25(-1)}$ (+.5)

(+1) (+.5)

$= \frac{595+315i}{74} = \frac{595}{74} + \frac{315}{74}i \approx 8.04 + 4.26i$

5. Let f be defined by the graph below.



(a) [2] (RationalActivity #2) Is f a function? Why or why not? (+1)

(+1) yes? passes the vertical line test

i.e. each x value corresponds to at most one y value

(b) [2] (WebHW6 #11) Is f a polynomial? Why or why not?

(+1) no polynomials do not have jumps or holes (+1)

(c) [2] (RationalActivity #2) Estimate the domain of f.

x-values (+.5) $x \neq -1$ or 2 (+.5) (+.5) notation (+.5) OR $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

(d) [1] (WebHW8 #7) Complete the following statement: as $x \to \infty$, $y \to$ 0 (+1)

(e) [1] (RationalActivity) Identify any vertical asymptotes.

@ $x=-1$ (+1)

(f) [1] Estimate $f(-2)$

$= -1$ (+1)

(g) [1] Estimate all possible x such that $f(x) = 1$.

@ $x=0$ b/c $f(0)=1$ (+1)

(h) [4] (WebHW8 #9) Find the formula for f.

looks like $\frac{1}{x}$ (+1) but shifted left 1 unit (+.5) $\Rightarrow \frac{1}{x+1}$ (+.5)

Start (+.5)

rational function (+.5)

has a hole @ $x=2$ (+.5) $\Rightarrow \frac{1}{x+1}\frac{(x-2)}{(x-2)}$ (+.5)

so $f(x) = \frac{(x-2)}{(x+1)(x-2)}$

note $f(-2) = \frac{-2-2}{(-2+1)(-2-2)} = \frac{-4}{4} = -1$ so matches (f)

(i) [3] (EndPolynomialActivity #8) Sketch a graph of $-f(x) - 1$.

order (+.5)

shape (+.1)

asymptote (+.5)

mult the y-coord by -1 (+.5)

shift vert down by 1 (.5)

final graph is dashed

8. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (Exam2 Aut2016 #9) When Pokemon-Go users begin the game they are given 25 Poke balls to use to catch pokemon. To get more Pokeballs, the user must walk or travel. On average one can earn a Poke ball every 0.5 miles.

i. [4] Find a function describing the average number of miles a user will travel for a Pokeball. Be sure to clearly define any variables you create!

ii. [2] Find and justify the domain for your answer above.

(b) (WordProblems2 #8) It takes about \$28.50 of materials to produce a shoe. The designing fees for a shoe run about \$20,000. The first 5000 shoes are given to stores for displays and celebrities to wear.

i. [4] Write a function f describing the average cost of a shoe. Be sure to clearly define any variables you create!

ii. [2] How many shoes need to be made to bring the average cost of a shoe under \$40?

(a)

let m = # of miles walked (+1)

i) Ave # of miles for Pokeballs $= \dfrac{\text{total \# of miles}}{\text{total \# of Pokeballs}}$ (+1)

$= \dfrac{m}{\text{free Pokeballs} + \text{earned Pokeballs}}$ (+1)

start (+1.5)

$= \dfrac{m}{25 + \frac{1 \text{ Pokeball}}{.5 \text{ miles}} \cdot m \text{ miles}}$

$= \dfrac{m}{25 + 2m}$ (+.5)

ii) $0 \le m$ (+1) or $[0, \infty)$

(+1) [It's possible to travel positive distances not negative.

Note the denominator $= 0$ when

$25 + 2m = 0$

$\Rightarrow 2m = -25$

$\Rightarrow m = -12.5$

if do this +1

So $m \neq -50$, but since m needs to be positive we dont need to worry about the rest. asy.

(b)

let x be the # of shoes made (+1)

i) Ave cost of a shoe $= \dfrac{\text{total Costs of making shoes}}{\text{\# of shoes sold}}$ (+.5)

$= \dfrac{\text{design cost} + \text{materials cost}}{\text{shoes made} - \text{shoes giveaway}}$ (+1) (+1)

start (+.5)

$= \dfrac{20{,}000 + 28.50 \cdot x}{x - 5000}$

note: the answer could look different if you choose a different variable.

ii) find x so that

(+1) $40 = \dfrac{20{,}000 + 28.50x}{x - 5000}$

$40(x - 5000) = 20{,}000 + 28.50x$

$40x - 20{,}000 = 20{,}000 + 28.50x$

$-28.5x \quad +20{,}000 \quad +20{,}000 \quad -28.5x$

alg (+1) $\dfrac{11.5x}{11.5} = \dfrac{40{,}000}{11.5}$

$x = \dfrac{40{,}000}{11.5} \approx$

6. Provide a graph AND an algebraic rule/expression for each of the functions described below:

(a) [3] (Quiz2 #3) A cubic polynomial whose *only* roots are -2 and 3.

There are so many answers that can work?

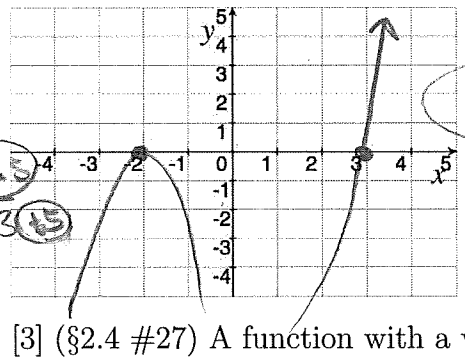
$y = (x--2)^2(x-3)$

No more roots (+.5)

(+.5) [-2 is a root => $(x-2)$ is factor
3 is a root => $(x-3)$ is factor

(+.5) 3rd degree

Match (+.5)
shape of graph (+.5)
only roots are -2 & 3 (+.5)



(b) [3] (§2.4 #27) A function with a vertical asymptote at $x = 4$.

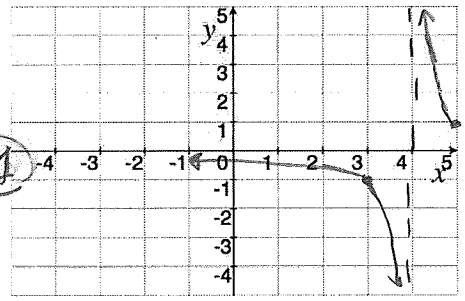
There are so many answers that can work?

$1/x$ shifted right 4

=> $\frac{1}{x-4}$

vert asy (+1)

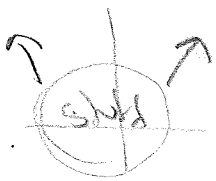
match (+1)
vert asy @ $x=4$ (+1)



7. The area of a rectangle is $3x^4 - 6x^3 + 17x^2 - 10x + 20\text{cm}^2$.

(a) [2] (§2.2 #38) Describe the end behavior of the function that returns the area of the rectangle given an x.

(+.5) fourth degree polynomial => both ends point in same direction
(+1) leading coef. is positive => as $x \to \infty$, $y \to \infty$
and as $x \to -\infty$, $y \to \infty$



start (+.5)

(b) [1] (Quiz2 #2) If $x = 1$, find the area of the rectangle.

Area $= 3(1)^4 - 6(1)^3 + 17(1)^2 - 10(1) + 20$ cm^2 plug in 1 (+.5)

$= 3 - 6 + 17 - 10 + 20$ cm^2

$= -3 + 7 + 20$ cm^2 $= 24$ cm^2 get it (+.5)

(c) [4] (WebHW7 #14) Find the rectangle's width (as a function of x!) if the length is $x^2 - 2x + 4$ cm. start (+.5)

Area = width · length (+.5)

=> width $= \frac{\text{Area}}{\text{length}}$ (+.5)

$= \frac{3x^4 - 6x^3 + 17x^2 - 10x + 20 \text{ cm}^2}{x^2 - 2x + 4 \text{ cm}}$

$= 3x^2 + 5$ (+.5) (+.5)

set up (+.5)

algorithm (+1)

$$\begin{array}{r} 3x^2 \quad +5 \\ x^2-2x+4 \overline{\big) 3x^4 - 6x^3 + 17x^2 - 10x + 20} \\ -(3x^4 - 6x^3 + 12x^2) \\ \hline 5x^2 - 10x + 20 \\ -(5x^2 - 10x + 20) \\ \hline 0 \end{array}$$