Exam 1 **TMath 115** Winter 2019

NAME: Key

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T (F) $\frac{1}{b+1} + \frac{1}{b} = \frac{3}{b+1}$

$b\frac{1}{b+1} + \frac{1}{b}\frac{b+1}{b+1} = \frac{b+b+1}{b(b+1)} = \frac{2b+1}{b(b+1)}$

(T) F $f(x) = -2(x+1)^2 - 7$ has a vertex at $(-1, -7)$

$-2(x-{}^{-}1)^2 - 7$ $a(x-h)^2 + k$ @ (h,k)

Show all your work. Reasonable supporting work must be shown to earn credit.

2. [3] Let f be a function with 2 in its domain. Explain what the notation $f(2)$ means as you would to a 12 year old.

$f(2)$ is the output from the function f when 2 is given (+.5)

That is, there is a machine called f

started (+.5)

and $f(2)$ is what is returned (+1) when 2 is put into (+1) the machine.

order of op (+.5)
function in frac (+1)
common den (+.5)
addition (+.5)
simplify (+.5)

3. [3] Perform and simplify $\frac{x-2}{\frac{x}{3}} + \frac{x}{5}$

$\frac{3}{3}\frac{x-2}{\frac{x}{3}} + \frac{x}{5}$

$\frac{3(x-2)}{x} + \frac{x}{5}$

$\frac{15(x-2)+x^2}{5x}$ or $\frac{x^2+15x-30}{5x}$

OR $x-2 \div \left(\frac{x}{3}\right) = \frac{x-2}{1} \cdot \frac{3}{x} = \frac{3(x-2)}{x}$

$\frac{5}{5}\cdot\frac{3(x-2)}{x} + \frac{x}{5}\cdot\frac{x}{x}$

$\frac{15(x-2)}{5x} + \frac{x^2}{5x} = \frac{x^2+15x-30}{5x}$

4. [4] (WebHW1 #8) The distance that a spring will stretch varies directly as the force applied to the spring. A force of 60 pounds is needed to stretch a spring 6 inches. What force is required to stretch the spring 10 inches?

start (+.5)
variable def (+.5)

dist stretch $= K$ (force applied) (+1)

6 inch $= K$ (60 lbs)

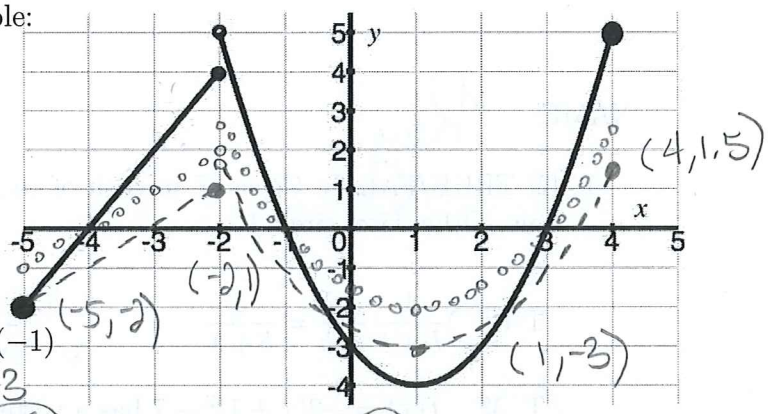
(+.5) $\Rightarrow K = \frac{1}{10}$ inch/lbs

find 10 in $\frac{1 \text{ in}}{10 \text{ lb}}$ (force to stretch 10 in) (+1)

$\Rightarrow \frac{10 \text{ lbs}}{\text{in}} \cdot 10 \text{ in} =$ force want

$\Rightarrow$ 100 lbs (1.5)

5. Let f be the piece-wise defined function comprised a line and a parabola shown below.



(a) Estimate the following *if* possible:

i. [1] (§1.3 #60) $f(-1)$

0

ii. [2] (§1.6 #10) $(f+f)(4)$

$= f(4) + f(4)$ +1

$= 5 + 5 = 10$ +1

iii. [2] (WebHW4 #15) $(f \circ f)(-1)$

$f(f(-1)) = f(0) = -3$

+.5 +.5 +.5

graph reading +.5

iv. [2] (Transformations Activity #5) the range of f.

y-values/outputs +1 $[-4, 5]$ values +.5 end points +.5 graph read +.5

v. [1] (Transformations Activity #5) all possible x such that $f(x) = 0$.

Should be worth 2 points

$x = -4, -1$ and 3 all work / got one +.5

+.5 +.5 +.5

(b) [4] (115ExamAut2016 #3) Find the formula for f in the indicated form:

$$f(x) = \begin{cases} 2x+3 & \text{if } -5 \le x \le -2 \\ & \text{if } -2 < x \le 4 \end{cases}$$

+.5 $y = mx + b$ line

+.5 slope $= \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

+.5 thru $(-4, 0)$ so $0 = 2(-4) + b$

+.5 so $b = 3$

$\Rightarrow y = 2x + 3$

quadratic $y = a(x-h)^2 + k$ +.5

vertex @ $(1, -4)$

$\Rightarrow y = a(x-1)^2 - 4$] +1

thru $(0, -3)$ so $-3 = a(0-1)^2 - 4$

so $1 = a$

$\Rightarrow y = (x-1)^2 - 4$] +.5

(c) [4] (WebHW4 #4) Graph $\frac{1}{2}f(x) - 1$.

+.5 first implement and +1

+.5 mult the y value by ½ / vertical stretch by ½ implemented +1

Second +.5 shift down 1 unit implemented +1

final answer

Shape +1

both pieces +1

6. Let h be the *function* defined by: $h(x) = \begin{cases} \frac{1}{2}x - 2 & -4 \le x \le 2 \\ 2x - 5 & 2 < x < 4 \end{cases}$

(a) [2] (FunctionActivity #1)
Find $h(3)$ use 2nd line (+1)

$2(3)-5 = 6-5 = 1$ (+1)

(b) [2] (WebHW2 #7)
Find the y-intercept of h
when $x=0$ (+1)
first line (+.5)

-2

(c) [1] (WebHW2 #19)
What is the domain of h?

$-4 \le x < 4$ graphical +.5

or $[-4, 4)$ inputs (+.5) 4's (+.5)

(d) [3] (WebHW3 #5) Graph h on the axes.

(+1) (+1)

endpoints (+1)

7. Provide a graph AND an algebraic rule/expression for each of the functions described below:

(a) [3] (WebHW5 #8) A quadratic with a maximum at $(2, 4)$.

match (+.5)

shape (+.5)

vertex & max (+.5)

$y = a(x-h)^2 + k$] (+.5) vertex @ $(2,4)$

$\Rightarrow y = a(x-2)^2 + 4$] (+.5)

upside down] (+.5)

$y = -(x-2)^2 + 4$

(b) [3] (§1.2 #76) A line perpendicular to $y = 3x - 2$. $y = mx + b$ (+.5)

match (+.5)

Shape (+.5)

slope is correct (+.5)

$\perp$ to slope of 3

$\Rightarrow$ slope $= -\frac{1}{3}$ (+.5)

note there are many correct answers here?

$y = -\frac{1}{3}x$ works] (+.5)

8. [2] Explain one concept that you studies well while preparing for this test but don't feel as if you got to fully demonstrate. (~~Note that I~~ am not asking for an analysis of what the test is lacking but rather for a stunning display on your part of information/skills that you know!) (+1) introduce topic

(+1)

9. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

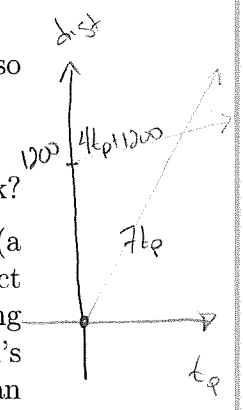
(a) (Quiz1 #4) You have 16 oz of mocha that is 10% espresso sitting in a 20 oz cup. You would like to add x oz of espresso to your drink.

i. [3] Write a function that returns the percentage (in decimal form) of espresso in the cup.

ii. [1] What is the domain of the function you defined above?

iii. [1] Is it possible to add enough espresso to have a 20% espresso mocha drink?

(b) [5] (WordProblems #3) Seismic waves travel at about 4km/s but Megan has (a really fast!) carrier pigeon that travels 7km/s. Assume that Megan's first instinct when feeling a quake is to "tweet" the experience and that process (recognizing it's an earthquake, finding her carrier pigeon, attaching a message to the bird's leg, and the message being broadcast) takes 5 minutes. How far does a Megan follower have to be from Megan to know there is an earthquake before feeling it? (inspired by http://xkcd.com/723)



(a)

i) % of espresso = (+1) $\frac{\text{espresso have + espresso add}}{\text{original mocha + added liquid}}$ (+.5)

start (+.5)

$= \frac{16 \cdot .10 + x}{16 + x} = \frac{1.6 + x}{16 + x}$ (+.5) (+.5)

ii) (+.5) $0 \le x \le 4$ oz (+.5)

↳ more would make the cup over flow

↳ hard to pull out espresso for mixture

iii) (+.5) $.2 = \frac{1.6 + x}{16 + x}$

$.2(16 + x) = 1.6 + x$

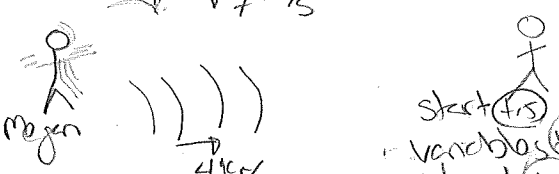
$3.2 + .2x = 1.6 + x$

$1.6 = .8x$

$\Rightarrow x = 2$ oz so yes (+.5)

or upper bound when (+.5) $x = 4$ so upper % is $\frac{1.6 + 4}{16 + 4} = \frac{5.6}{20}$ $= 28\%$ which is bigger than 20% so yes (+.5)

(b)



want to find when

start (+.5)

variables (+.5)

$d = r \cdot t$ (+.5)

(+.5) [dist of pigeon > dist of earthquake waves]

$7\,\text{km/s} \cdot (\text{pigeon flying time})\,s > 4\,\text{km/s}\,(\text{time since earthquake})\,s$

(+1) note pigeon flying time s = time since earthquake $s - 5\,\text{min}\left(\frac{60\,s}{1\,\text{min}}\right)$

$= -300\,s$

$\Rightarrow$ pigeon flying time = time since earthquake

(.5 x 5) $7(t_e - 300) > 4t_e$

$7t_e - 2100 > 4t_e$

$3t_e > 2100$

$t_e > 700$ (+1)

a follower needs to be more than $4(700)$ or 2800 km away. (answer +.5)