Exam 1 **TMath 115** Spring 2016

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NAME: Key

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

   T (F) $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$ $\neq \frac{b+a}{ab}$

   T (F) $y = x^{\frac{1}{2}} + 5x - 5$ is a polynomial. $x^{1/2} = \sqrt{x}$ polynomials have positive whole number exponents

   T (F) If $y = -2x^4 - 56x^3 + 70x^2 - 81$, when $x \to \infty$, then $y \to \infty$.

   T (F) The function $y = \frac{(x-2)^2}{x-2}$ has an asymptote at $x = 2$. as $x \to \infty$ $y \to -\infty$ $\frac{(x-2)^2}{x-2} = \frac{(x-2)(x-2)}{x-2} = x-2$ with hole @ $x=2$

   (T) F The function $y = \frac{(x-2)^2}{x-2}$ equals $y = x - 2$ except at $x = 2$

   (T) F If $f(2) = 3$, then $f^{-1}(3) = 2$.

   (T) F In this class a 70% corresponds to earning a 2.0.

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (WebHW17 #8) Simplify $\left(\frac{-6x^5z}{y^3}\right)^3\left(\frac{x^2}{z}\right)$

$(x^5)^3 = x^5 x^5 x^5 = (xxxxx)(xxxxx)(xxxxx) = x^{15}$

$x^{15} x^2 = (xxx\,xx\,\ldots x)(xx) = x^{17}$

$\frac{(-6x^5z)^3}{(y^3)^3}\left(\frac{x^2}{z}\right) = \frac{(-6)^3(x^5)^3 z^3}{(y^3)^3} \cdot \frac{x^2}{z} = \frac{-216x^{15}z^3x^2}{y^9 z}$ (+1) (+.5) (+1)

$= \frac{-216x^{17}z^2}{y^9}$ (+1)

got A (+1.5)

$\frac{36}{6}$

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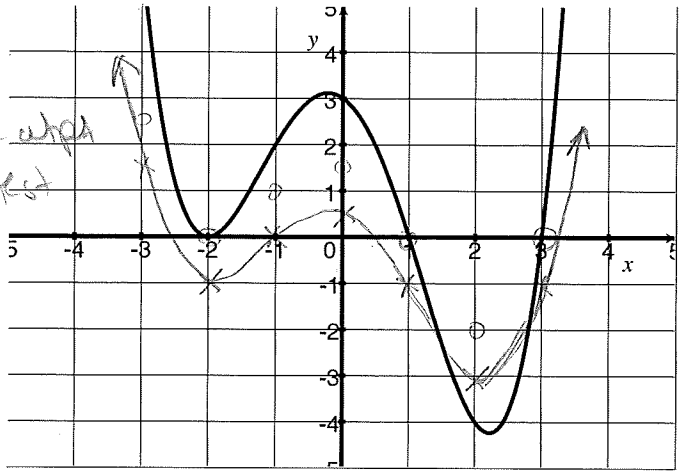
3. Let α be the graph shown on the right



(a) [1] Is α a function?

yes (+1)

every input has at most 1 output

or it passes the vertical line test

(b) [1] (Review §1.6 #55) Estimate $\alpha(0)$.

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(c) [2] (InverseWks #3) Does α have an inverse function? Why or why not?

(+1) no b/c it fails the horizontal line test (+1)

or both -2 and 1 are sent to 0 so $\alpha^{-1}(0) = -2$ AND 1

(d) [2] (Quiz3 #3) Could α be a degree three polynomial? Why or why not?

(+1) no b/c of end behavior, both ends go up implying the graph is an even degree polynomial (+1)

(e) [4] (Polynomial2Wks #2) Find the rule for α

(+1) roots @ $-2, 1, 3 \Rightarrow (x+2), (x-1)$, and $(x-3)$ are factors

(+1) touches but does not cross @ $x=-2 \Rightarrow (x+2)^2$ is a factor

crosses @ $x=1$ and $x=3 \Rightarrow (x-1)$ and $(x-3)$ are factors

$\alpha(x) = a(x+2)^2(x-1)(x-3)$ (+.5)

passes thru $(0,3)$ so $3 = a(0+2)^2(0-1)(0-3)$ (+.5)

$\Rightarrow 3 = a4\cdot(-1)(-3) \Rightarrow 3 = 12a$

$\Rightarrow a = 1/4$ (+.5)

$\alpha(x) = \frac{1}{4}(x+2)^2(x-1)(x-3)$ polynomial (+.5)

(f) [3] (Review §1.5 #51) Graph $\frac{1}{2}\alpha(x) - 1$ on the right axis above.

1) vertical stretch by $\frac{1}{2}$ ie. multiply y by $\frac{1}{2}$ +.5

2) vertically move down 1 unit +.5

order +.5

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4. Let $f(x) = \dfrac{3x+7}{x+2}$

(a) [3] (WebHW13 #4) Find the quotient and remainder when performing the division. Check your work by verifying that (Quotient)(Divisor)+Remainder=Dividend

$$\begin{array}{r} 3 \quad (+.5) \\ x+2 \overline{)\,3x+7} \quad R1 \quad (+.5) \\ -(3x+6) \\ \hline 1 \end{array}$$

$$\frac{3x+7}{x+2} = 3 + \frac{1}{x+2}$$

or

$$3x+7 = 3(x+2)+1$$

set up (+1)
algorithm (+1)

(b) [1] (§2.5 #40) Identify any vertical asymptotes.

Domain: $x+2 \neq 0$, $x \neq -2$ (+.5)

so vert asy @ $x = -2$ (+.5)

(since $f(x)$ is already reduced)

(c) [4] (WebHW16 #9) Given that f is one-to-one or that f has an inverse, find $f^{-1}(x)$.

clear den (+.5)
algebra (+1.5)

$y = \dfrac{3x+7}{x+2}$

(+1) $x = \dfrac{3y+7}{y+2}$

$x(y+2) = 3y+7$

$xy+2x = 3y+7$

$-xy \quad -xy$

$2x = 3y - xy + 7$

$-7 \quad -7$

$2x-7 = 3y-xy$

for one side (+.5)
only one y on one side (+.5)

$2x-7 = y(3-x)$

$y = \dfrac{2x-7}{3-x}$

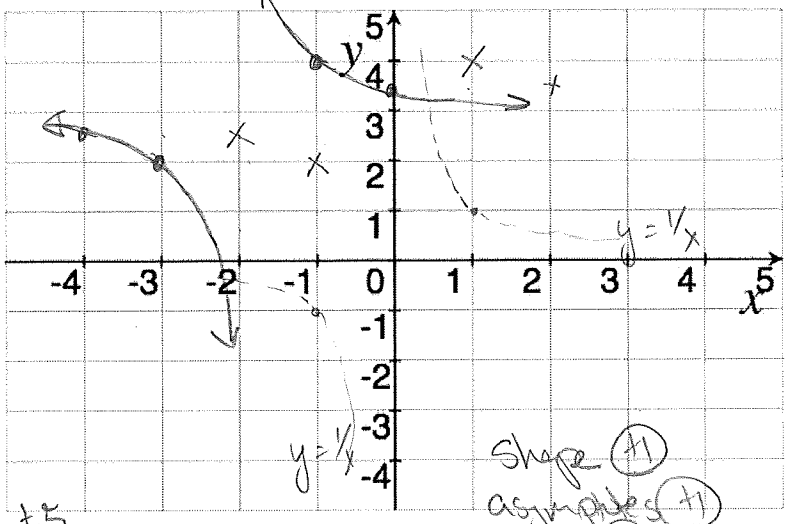
5. Consider the function $g(x) = \frac{1}{x+2} + 3$

(a) [1] Find the domain of g.

$x+2 \neq 0$

$x \neq -2$

(b) [3] (Rational Wks #3) Graph g.



notice $g(x)$ is the graph of $y = \frac{1}{x}$ but

1) shifted up by 3
2) shifted left by 2

x x +.5
+.5 3

shape (+1)
asymptotes (+1)
got it (+1)

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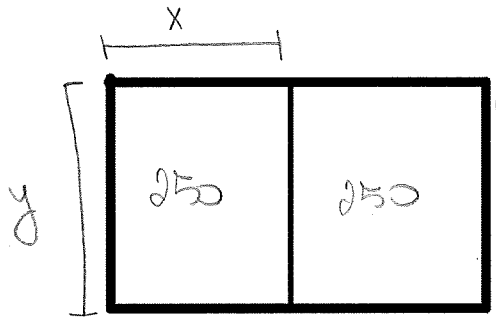
6. [5] (pg 195 #77) A farmer wants to construct a fence around a total of 500 square feet in the following configuration where each pen contains 250 square feet. The outer boundary of the pens require a heavy fencing material that costs \$4.5 per foot and the internal partitions cost \$2 per foot. Find a function that describes the total cost of the fence in terms of only one variable.

labels (+1)



outside fence costs \$4.5/ft
inside fence costs \$2/ft

(+.5) Cost = Outside fence cost + Inside fence cost
(+.5) $= 4.5\,\$/ft\,(\text{outside fence}) + 2(\text{inside fence})$
$= 4.5(y + x + x + y + x + x) + 2(y)$
$= 4.5(4x + 2y) + 2y$
$= 18x + 9y + 2y$
(+1) $= 18x + 11y$
sub (+.5) $= 18\left(\frac{250}{y}\right) + 11y$

note $x \cdot y = 250$
$\Rightarrow x = \frac{250}{y}$ (+1)

7. [5] (Presenations #2) You have \$5500 in a retirement fund and would like a return of 5.5% (to do slightly better than the historical trends of inflation). There are 5 year CDs (certificate of deposits) being offered with an annual rate of 2.15% and index funds (a collection of stocks from companies included in measures like the S&P 500) that returned 8.2% since the 1990's (Thomson Reuters, 2010 S&P 500 Composite Index total return for the period 12/31/1989 to 12/31/2009). How much money do you relegate to a CD and how much money do you put in an index fund to get an annual return of 5.5% for the next five years?

labels (+1)

x = \$ into CD funds
y = \$ into index funds

\$ want = \$ from CD + \$ from index

$5500 \cdot .055 = x \cdot .0215 + y \cdot .082$
(+1) (+.5) (+.5)

note $x + y = 5500$ (+.5)
$\Rightarrow x = 5500 - y$
sub (+.5)

$5500 \cdot .055 = (5500 - y) \cdot .0215 + y \cdot .082$
$302.5 = 118.25 - .0215y + .082y$
$-118.25 \quad -118.25$
$184.25 = .0605y$

solve for $x + y$ (+1)

$\Rightarrow y = 3045.45$ into index
$\Rightarrow x = 2,454.55$ in CDs

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