Exam 2 # TMath 115 /45 Autumn 2016

NAME: Key

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

   [✓ T] F x is a polynomial.

   T [✓ F] $2^x - 5x^2$ is a polynomial. x is an exponent

   T [✓ F] $\frac{x}{3x^{-2} + 4\sqrt{x}}$ is a rational function. $3x^{-2}+4\sqrt{x}$ is not a polynomial

   [✓ T] F $(x-2)$ is a factor of $y = 2x^3 - 4x - 8$.. $x-2$ is a factor if and only if 2 is a root $2(2)^3 - 4 \cdot 2 - 8 = 16 - 8 - 8 = 0$

   [✓ T] F Let f have an inverse, then $(f(f^{-1}))(130) = 130$. f^{-1} undoes f

   T [✓ F] $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$ $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (§1.7 #56) Solve for y,

$$4x = \frac{y+2}{3y+1}$$

$(3y+1)4x = \frac{y+2}{\cancel{3y+1}} \cancel{3y+1}$

$4x(3y+1) = y+2$

$12xy + 4x = y + 2$
$\quad -y \quad -y$

$12xy - y + 4x = 2$
$\quad -4x \quad -4x$

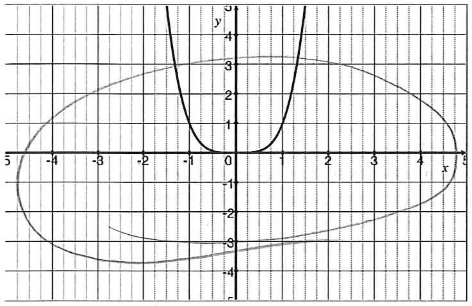
$12xy - y = 2 - 4x$

$y(12x - 1) = 2 - 4x$

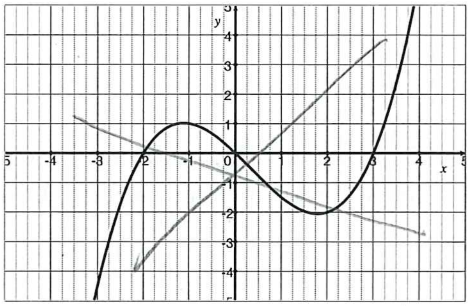
$y = \frac{2-4x}{12x-1}$

Start (+.5)
~~clear den~~ (+.5) fractions (+.5)
respect both sides & eq (+.5)
dist (+.5)
factored (+1)
got y's on one side (+.5)
got y alone (+.5)

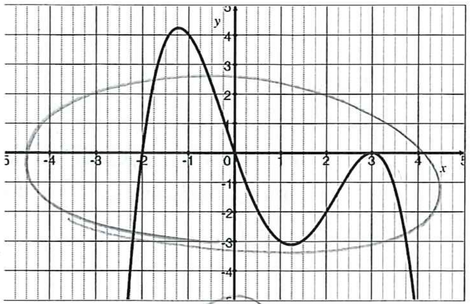
3. [3] (WebHW12 #4) Identify all of the graphs below that could be a 4th degree polynomial.



(+1)

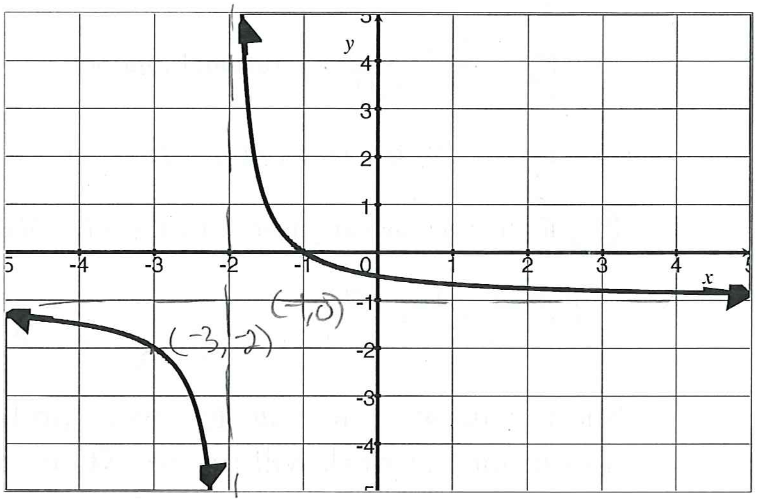


(+1)



(+1)

4. Let q be the function graphed below:



(a) [2] (WebHW14 #11) Find the range of q

(+1) y-values
(+1) got it

$y \neq -1$ or

$(-\infty,-1)\cup(-1,\infty)$

(b) [2] (WebHW11 #7) Could q be the graph of a polynomial? Why or why not?

No - there is a break in the graph

(+1) (+1)

if true +.5

(c) [2] (InverseWks #3) Note, q has an inverse. Identify a point on the graph of q^{-1}

Swap x's and y's from points on graph of q (+1)

So (0,-1) (-2,-3), etc (+1)

(d) [2] (WebHW14 #6) Describe the graph transformations needed to transform $p(x) = \frac{1}{x}$ into the graph of q.

Shift LEFT two units (+1)

and DOWN one unit (+1)

(e) [2] Find the algebraic rule for q.

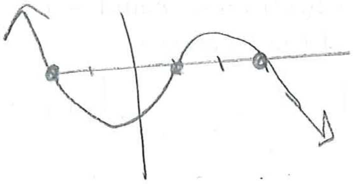
form (+.5)

$$q(x) = \underbrace{\frac{1}{x+2}}_{(+1)} \underbrace{-1}_{(+.5)}$$

$$= \frac{1}{x+2} - \frac{x+2}{x+2} = \frac{-x-1}{x+2}$$

5. [3] (Quiz3 #2) Write a polynomial p that satisfies the following criteria:

- as $x \to \infty$, then $y \to -\infty$ (+1)
- -2, 1, and 3 are the only roots. (+1)



Note: there is more than one right answer!

$-1(x - -2)(x-1)(x-3) = y$

Polynomial (+1)

6. [4] (§1.7 # 56) The function $g(x) = \dfrac{2}{x-5} - 1$ is one-to-one (i.e. has an inverse). Find g^{-1}.

$y = \dfrac{2}{x-5} - 1$

(+1) $x = \dfrac{2}{y-5} - 1$

+1 +1

$x + 1 = 2/y-5$

$(y-5)\, x + 1 = \dfrac{2}{y-5}$ ~~y-5~~

$(y-5)(x+1) = 2$

$y - 5 = \dfrac{2}{x+1}$

$y = \dfrac{2}{x+1} + 5$

$xy + y - 5x - 5 = 2$

$xy + y = 2 + 5x + 5$

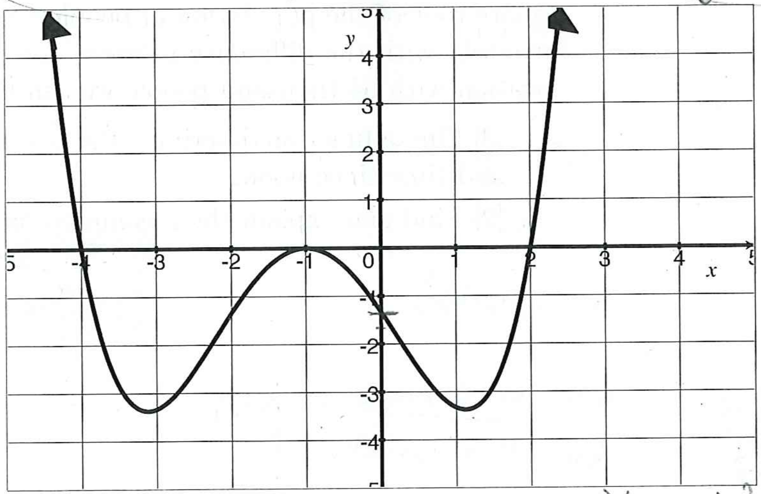
$y(x+1) = 7 + 5x$

$\Rightarrow y = \dfrac{7+5x}{x+1}$

clear den (+.5)
respect both sides (+.5)
order of op (+1)

alg (+1)

7. Let p be the polynomial graphed below.



(a) [1] True or [✓ False]
The polynomial p has odd degree.

end behavior has both sides going up
$\Rightarrow$ even degree

(b) [4] (§2.3 #40) Assume when p is completely factored, each real zero corresponds to a factor of the form $(x - c)^m$.
Find the equation of least degree for p.

(+1)
-4 is a root $\Rightarrow x - -4$ is a factor
-1 is a root $\Rightarrow x - -1$ is a factor
2 is a root $\Rightarrow x - 2$ is a factor

(+1)
crosses @ $-4 \Rightarrow (x - -4)$ is a factor
touches @ $-1 \Rightarrow (x - -1)^2$ is a factor
crosses @ $2 \Rightarrow (x - 2)$ is a factor

So $y = a(x+4)(x+1)^2(x-2)$

(+.5) passes thru $(0, -1.3)$ so

(+.5) $-1.3 = a(0+4)(0+1)^2(0-2)$

$-1.3 = -8a$ alg (+.5)

$\Rightarrow a = .1625$

$y = .1625(x+4)(x+1)^2(x-2)$ (+.5)

8. (Quiz3 #3) [5] The area of a rectangle is $5x^4 - 15x^3 + 22x^2 - 6x + 8$ cm^2. Its length is also a function of x and is $x^2 - 3x + 4$ cm. If the length is 2 cm, what are the possible widths of the rectangle?

$x^2-3x+4=2$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$x-2=0$ $x-1=0$

$x=2$ or $x=1$

(+2)

(+3)

Area is

$5(2)^4-15(2)^3+22(2)^2-6(2)+8$

$=44$

or

$5(1)^4-15(1)^3+22(1)^2-6(1)+8$

$=14$

So width is $\frac{14}{2}=$ ⑦ or $\frac{44}{2}=$ (22)

Area = width · length

=> width = $\frac{\text{Area}}{\text{length}}$

or

$$x^2-3x+4\overline{)5x^4-15x^3+22x^2-6x+8}$$

quotient: $5x^2+2$

$-(5x^4-15x^3+20x^2)$

$2x^2-6x+8$

$-(2x^2-6x+8)$

0

So width is

$5(2)^2+2=22$ or $5(1)^2+2=7$

9. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (ModelingWks #2) When Pokemon-Go users begin the game they are given 25 Poke balls to use to catch pokemon. To get more Poke balls, the user must walk or travel. On average one can earn a Poke ball every 0.7 miles. 1 pokeball for .7 miles

i. [3] Find a function describing the average number of miles a user will travel for a Poke ball. =>

ii. [2] Find and explain the meaning of any vertical asymptotes.

(b) (WordProblems2 #3) The daily pokemon population varies directly with the square root of the population of people (measured in thousands) in the area and inversely with the difference between the current time and noon. At 10am in a location with 64 thousand people we can find 5 pokemon.

i. [3] Find a function describing the pokemon population as a function of people and time since noon.

ii. [2] Find and explain the meaning of any vertical asymptotes.

Start (+.5)

define variables

(+1)

a) i) m = # of miles traveled

ave # of miles for pokeball $=$ $\frac{\text{total \# of miles}}{\text{total \# of pokeballs}}$ (+.5)

$= \frac{m}{25 + \text{earned pokeballs}}$

$= \frac{m}{(+.5)\ 25 + \frac{1\text{ pokeball}}{.7\text{ miles}} \cdot m\ \text{miles}}$ (+.5)

$= \frac{m}{25+\frac{10}{7}m} = \frac{m}{25+1.42m}$

(+1)

(+.5) ii) { vert asy when $0=25+\frac{10}{7} \Rightarrow m=-17.5$

Domain/miles should stay >0 => so no vert asy

b) i) p = pop of people (in thousands)

t = time since noon

(+1) pokemon pop $= \frac{K\sqrt{p}}{t}$ (+.5) [when t = 2, p = 64, pokemon = 5

$5 = \frac{K\sqrt{64}}{2} \Rightarrow 10 = K8$

$\Rightarrow K = 1.25$

So pokemon pop $= \frac{1.25\sqrt{p}}{t}$

(+1) ii) vert asy when $t=0$ so at noon

(+1) At noon the pokemon pop. gets very very large