

Rational Functions

A *rational function* r is a function of the form $r(x) = \frac{f(x)}{g(x)}$ where f and g are polynomials with $g(x) \neq 0$

1. Identify which if the following are rational functions:

$$f(x) = \frac{1}{x} - 1$$

$$g(x) = \sqrt[4]{x^2 + 4}$$

$$h(x) = \frac{x^2 - 4}{x - 2}$$

2. Identify which of the above are functions and then determine the domain.

The line $x = a$ is called a *vertical asymptote* of the graph of a function f if $f(x) \rightarrow \infty$ as $x \rightarrow a^+$ or as $x \rightarrow a^-$ or if $f(x) \rightarrow -\infty$ as $x \rightarrow a^+$ or as $x \rightarrow a^-$.

The line $y = k$ is a *horizontal asymptote* of the graph of a function f if $f(x) \rightarrow k$ as $x \rightarrow \infty$ or if $f(x) \rightarrow k$ as $x \rightarrow -\infty$.

If $f(x) = \frac{N(x)}{D(x)}$ is a rational function where $N(x)$ and $D(x)$ do not have a common factor and c is a real zero of $D(x)$, then the line $x = c$ is a vertical asymptote.

3. A company manufactures widgets. Fixed daily costs are \$2000 and it costs \$1.5 to produce each widget.

(a) Write the average cost \bar{C} of producing x widgets.

(b) Find and interpret $\bar{C}(100)$ and $\bar{C}(500)$.

4. Graph

$$\frac{2}{x-1}$$

$$\frac{-3x-4}{x+2}$$

