Practice TMath 115 Final

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite different than those appearing here.

Let f & g, be functions with inverses f^{-1} and g^{-1} respectively.

T F $(x+3)^2 = x^2 + 9$ $\quad (x+3)^2 = (x+3)(x+3) = x^2+3x+3x+9 = x^2+6x+9$

T F $(f \circ g)(x) = (g \circ f)(x)$ $\quad f(x) = x^2$, $g(x) = x+1$, $(f\circ g)(x) = (x+1)^2$ vs $(g\circ f)(x) = x^2+1$

T F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$ $\quad \frac{f}{g}(x) = \frac{x^2}{x+1}$ $\quad \frac{g}{f}(x) = \frac{x+1}{x^2}$

T F $\sqrt{(x^2)} = x$ for all real numbers x. note $\sqrt{(-1)^2} \neq -1$

T F If 2 is a root of g, then $g(2) = 0$. a root is the same as an x-intercept

T F $\ln \frac{x}{y} = \ln x - \ln y$ for all positive numbers x and y.

T F $\log(\log(10)) = 0$. $\quad \log(\cancel{\log 10}^{1}) = \log(1) = 0$

T F $f(f^{-1}(54)) = 54$ $\quad f^{-1}$ undoes f

T F

T F

Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. Find all x such that

$$2(5-(8-x)^2)^{-\frac{1}{2}} - 1 = 0$$

$$\frac{2}{\sqrt{5-(8-x)^2}} - 1 = 0$$

$$\frac{2}{\sqrt{5-(8-x)^2}} = 1$$

$$2 = \sqrt{5-(8-x)^2}$$

$$4 = 5-(8-x)^2$$

$$4 = 5-(64-16x+x^2)$$

$$4 = -x^2+16x-59$$

$$0 = -x^2+16x-63$$

$$x^2-16x+63 = 0$$

$$x = \frac{16 \pm \sqrt{16^2-4(1)(63)}}{2(1)}$$

$$= 7, 9$$

or

$$4 = 5-(8-x)^2$$

$$-5 \quad -5$$

$$-1 = -(8-x)^2$$

$$1 = (8-x)^2$$

$$\pm\sqrt{1} = 8-x$$

$$-8 \pm 1 = -x$$

$$7, 9 = x$$

2. Perform the opperation

$$\frac{\frac{2}{x^2}-x}{x-2}+\frac{3x-5}{(x+4)(x-4)}$$

$$\frac{\frac{2x^2}{x^2}-xx^2}{x^2(x-2)}+\frac{3x-5}{(x+4)(x-4)}$$

$$\frac{2-x^3}{x^2(x-2)}+\frac{3x-5}{(x+4)(x-4)}$$

$$\frac{(2-x^3)(x+4)(x-4)+(3x-5)(x^2)(x-2)}{x^2(x-2)(x+4)(x-4)}$$

$$\frac{1+3i}{-6i+2} = \frac{1+3i}{2-6i}\cdot\frac{2+6i}{2+6i}$$

$$\frac{2+6i+6i+18i^2}{4+\cancel{12i}-\cancel{12i}-36i^2}$$

$$=\frac{2+12i+18(-1)}{4-36(-1)}$$

$$=\frac{-16+12i}{40}=\frac{-16}{40}+\frac{12}{40}i$$

$$=\frac{-2}{5}+\frac{3}{10}i$$

3. Given $m(x)=\dfrac{2x+3}{x-5}$, and $n(x)=\sqrt{4x-8}$,

(a) The inverse to the function m exists. Find m^{-1}.

$y=\frac{2x+3}{x-5}$

switch x's & y's

$x=\frac{2y+3}{y-5}$

$x(y-5)=2y+3$

$xy-\cancel{5x}=\cancel{2y}+3$

$-2y+\cancel{5x}\ -\cancel{2y}\quad +5x$

$xy-2y=3+5x$

$y(x-2)=3+5x$

$y=\frac{3+5x}{x-2}$

(b) If $p(x)=3m(x+1)$, find the domain and rule of p.

$p(x)=3m(x+1)$

$=3\ \frac{2(x+1)+3}{(x+1)-5}=\frac{3[2x+2+3]}{x-4}$

Domain: $x\neq 4$ or $(-\infty,4)\cup(4,\infty)$

(c) Find the domain and rule of $m\circ n$.

$(m\circ n)(x)$

$=m(n(x))$

$=m(\boxed{\sqrt{4x-8}})$

$\frac{2\boxed{}+3}{\boxed{}-5}$ so $\frac{2\sqrt{4x-8}+3}{\sqrt{4x-8}-5}$

Domain: $4x-8\geq 0$ and $\sqrt{4x-8}-5\neq 0$

$4x\geq 8$ $\quad$ $4x-8\neq 25$

$x\geq 1/2$ $\quad$ $4x\neq 33$

$x\neq 33/4$

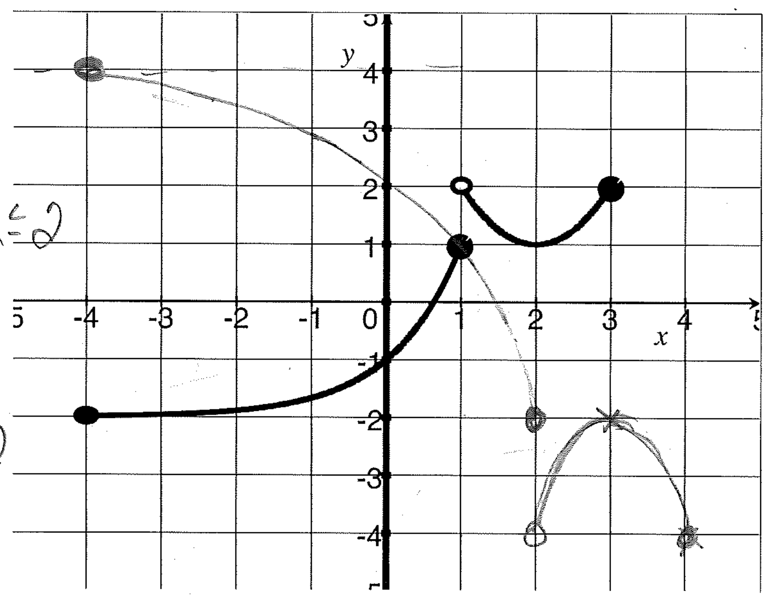
(d) Find the domain and rule of $\dfrac{n}{m}$.

$\left(\frac{n}{m}\right)(x)=\frac{\sqrt{4x-8}}{\left(\frac{2x+3}{x-5}\right)}$

Domain $4x-8\geq 0$ and $x-5\neq 0$ and $2x+3\neq 0$

$x\geq 1/2$ $\quad$ $x\neq 5$ $\quad$ $x\neq -3/2$

4. Let the following be the graph of g comprised of a parabola and an exponential function that have been shifted (not stretched).



(a) What is the domain of g?

X values

$[-4, 3]$ or $-4 \le x \le 2$

(b) What is the range of g?

y-values

$[-2, 2]$ or $-2 \le y \le 2$

(c) Use the graph above to *estimate* all x value(s) so that $g(x) = 1$?

i.e. find x when $y = 1$

@ $x = 1$ and $x = 2$

(d) Write down the piece-wise defined rule for g.

$g(x) = \begin{cases} \text{exp} & -4 \le x \le 1 \\ \text{parabola} & 1 < x \le 3 \end{cases}$

exp. function

$b^x + c = y$

looks like passes thru $(0, -1)$

So $b^0 + c = -1 \Rightarrow c = -2$

So $y = b^x - 2$

passes thru $(1, 1)$ so

$1 = b^1 - 2 \Rightarrow b = 3$

So $3^x - 2$

parabola

shifted up 1

shifted right 2

$(x-2)^2 + 1$

$g(x) = \begin{cases} 3^x - 2 & -4 \le x \le 1 \\ (x-2)^2 + 1 & 1 < x \le 3 \end{cases}$

(e) Draw the graph of $-2g(x - 1)$

mult the y-coord by -2

or vertical flip w/ vertical stretch by 2

horz shift right one unit

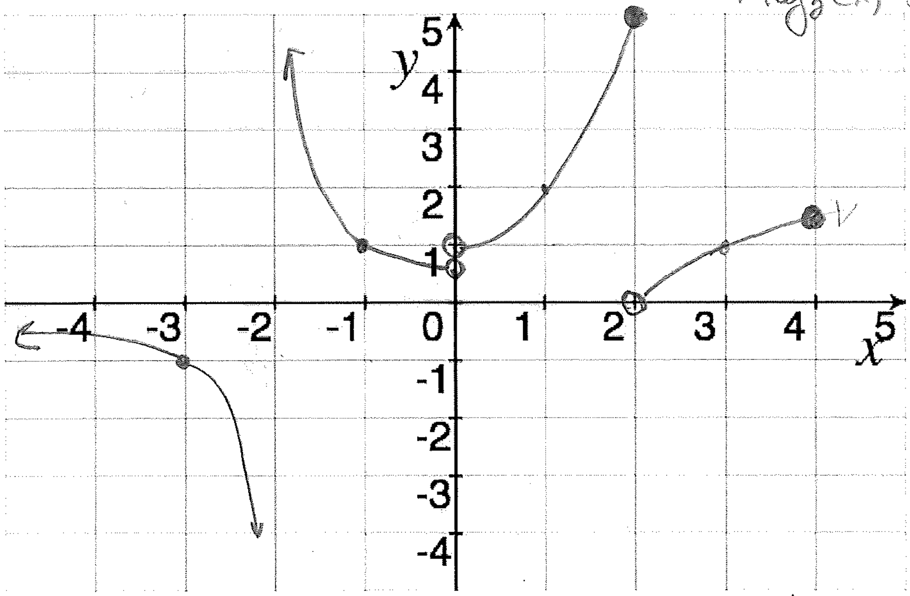
5. Define f by

graph $1/x$ shifted left 2 units

$$f(x) = \begin{cases} \frac{1}{x+2} & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x \le 2 \\ \log_2(x-1) & \text{if } 2 < x \le 4 \end{cases}$$

parabola shifted up 1

$\log_2(x)$ shifted right 1 unit



(a) Graph f on the axes above.

(b) Find the following if possible:

$f(1)$ $\quad 0 < 1 \le 2$ so use 2nd line

$(1)^2 + 1 = 2$

$\frac{4}{f(2)} + f(3) = \frac{4}{2^2+1} + \log_2(3-1)$

use 2nd line

use 3rd line

$= \frac{4}{5} + \log_2(2) = \frac{4}{5} + 1$

$= 9/5$

$f(0)$

does not exist

(not in the domain of f)

$f(\frac{-1}{4})$

$-1/4 < 0$ use 1st line

$\frac{1}{-1/4+2} = \frac{1}{7/4} = 4/7$

Domain of f

$x \neq -2, 0$ and $x \le 4$

or $(-\infty, -2) \cup (-2, 0) \cup (0, 4]$

$(x^2)^3 = x^2x^2x^2 = xxxxxx = x^6$

$x^2x^3 = (xx)(xxx) = x^5$

6. Find all of the exact values x that satisfy the following:

$5^{5x}25^{x^2} = 125$

$5^{5x} \cdot (5^2)^{x^2} = 5^3$

$5^{5x}5^{2x^2} = 5^3$

$\log_5 5^{5x+2x^2} = \log_5 5^3$

$5x+2x^2 = 3$

$2x^2+5x-3=0$

$x = \frac{-5 \pm \sqrt{25+4(2)(3)}}{4}$

$5^{4x-1} = 7^x$

$\ln 5^{4x-1} = \ln 7^x$

$(4x-1)\ln 5 = x\ln 7$

$4x\ln 5 - \cancel{\ln 5} = \cancel{x\ln 7}$

$-x\ln 7 + \cancel{\ln 5} \quad -x\ln 7 + \ln 5$

$x4\ln 5 x\ln 7 = \ln 5$

$x(4\ln 5 - \ln 7) = \ln 5$

$x = \frac{\ln 5}{4\ln 5 - \ln 7}$

7. Find all exact values for x that satisfy the following:

$\log(x-16) = 2 - \log(x-1)$

$\log(x-16) + \log(x-1) = 2$

$\log(x-16)(x-1) = 2$

$(x-16)(x-1) = 10^2$

$x^2 - 17x + 16 = 100$

$x^2 - 17x - 84 = 0$

$(x-21)(+4) = 0$

$\Rightarrow x = 21$ or $\cancel{-4}$

$\hookrightarrow$ domain prob

$\frac{15}{3+2\cdot 5^x} = 4$

$15 = 4(3+2\cdot 5^x)$

$\frac{15}{4} = \cancel{3} + 2\cdot 5^x$

$-3 \quad -\cancel{3}$

$\frac{15}{4} - \frac{12}{4} = 2\cdot 5^x$

$\frac{3/4}{2} = \frac{2\cdot 5^x}{2}$

$\rightarrow 3/8 = 5^x$

$\log_5 3/8 = x$

8. Assume c, d, and z are all greater than zero and simplify:

$\frac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}} = \frac{(c^2d^6)^{1/2}}{(4c^3d^{-4})^{1/2}}$

$= \frac{(c^2)^{1/2}(d^6)^{1/2}}{4^{1/2}(c^3)^{1/2}(d^{-4})^{1/2}}$

$= \frac{cd^3}{2c^{3/2}d^{-2}} = \frac{cd^3d^2}{2c^{3/2}}$

$= \frac{d^5}{2c^{1/2}}$

$2 - \log_5(25z)$

$2 - \log_5(5^2 z)$

$= 2 - (\log_5 5^2 + \log z)$

$= 2 - (2 + \log z)$

$= \cancel{2} - \cancel{2} - \log z$

$= -\log z$

9. Given $f(3) = 0$ find the other roots of $f(x) = x^4 - 3x^3 - 25x^2 + 75x$

3 is a root $\Rightarrow$ $x-3$ is a factor

$(x-3)\cdot ? = x^4 - 3x^3 - 25x^2 + 75x$

$? = \dfrac{x^4 - 3x^3 - 25x^2 + 75x}{x-3}$

So long division

$$\begin{array}{r} x^3 \qquad\qquad -25x \quad R_0 \\ x-3 \,\overline{)\, x^4 - 3x^3 - 25x^2 + 75x} \\ -(x^4 - 3x^3) \qquad\qquad\qquad \\ \hline -(-25x^2 + 75x) \\ \hline 0 \end{array}$$

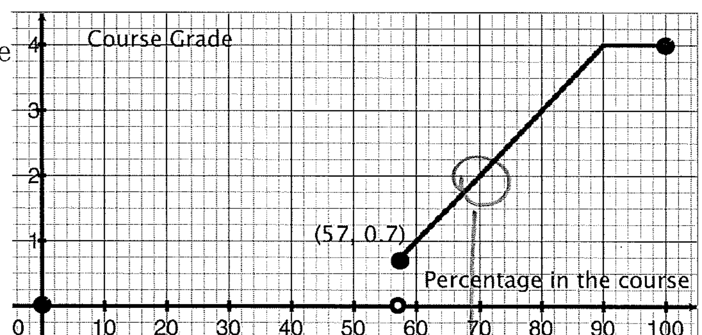
$x^3 - 25x = \dfrac{x^4 - 3x^3 - 25x^2 + 75x}{x-3}$

So $(x-3)(x^3-25x) = x^4 - 3x^3 - 25x^2 + 75x$ or $(x-3)x(x^2-25)$ or $(x-3)x(x+5)(x-5)$

10. Now that finals are next week, James T. Kirk would like to know if it is still possible to earn a 2.0. He has looked at the gradebook on MyMathLab and has computed the averages listed below.

Find what grade he needs to get on the final to receive a 2.0 in the course. In case you don't remember, the weights specified in the syllabus and the graph of the function f that takes your class percentage x and returns your score on a 4. scale are also provided.

	weight	James' ave
Mini-Quizzes	5%	95%
WebAssign	10%	10%
WrittenHW	15%	0%
Quizzes	15%	70%
2 Exams	30%	100%
Final	25%	



let f = final exam grade

to get a 2.0 he needs 70%

$5 \cdot .95 + 10 \cdot .10 + 15 \cdot 0 + 15 \cdot .70 + 30 \cdot 1.00 + 25 f = 70$

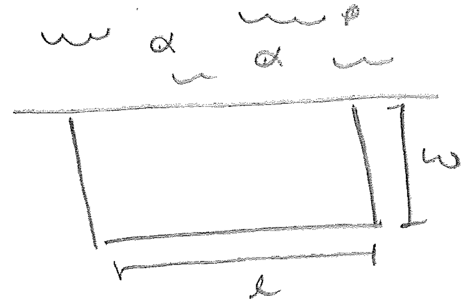
$46.25 + 25f = 70$

$-46.25 \qquad -46.25$

$\dfrac{\cancel{25}f}{\cancel{25}} = \dfrac{23.75}{25}$

$f = 95\%$

11. A rancher with 180 meters of fencing intends to enclose a rectangular region along a river (which serves as a natural boundary requiring no fence).



(a) Find the area of the region as a function of the width.

Area $= l \cdot w$

note $l + w + w = 180$

$l + 2w = 180$

$l = 180 - 2w$

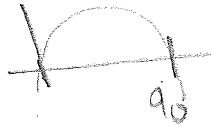
So Area $= (180 - 2w) \cdot w$

(b) Find the maximum area that can be enclosed.

Area $= (180-2w) \cdot w$

$= 180w - 2w^2$

$= -2w^2 + 180w$



x-coord $= \frac{-b}{2a} = \frac{-180}{2(-2)} = 45$

or

$-\frac{1}{2}y = w^2 - 90w$

$-\frac{1}{2}y + 45^2 = (w-45)^2$

$\Rightarrow y = -2(w-45)^2$

parabola opening down => we want the vertex

when $w = 45$ Area is 4050

12. Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope

Use $P_0 (\frac{1}{2})^{t/h} = P(t)$

start w/ P_0 when $t = 3$

$\frac{1}{5} P_0$ decay $\Rightarrow$ have $P_0 - \frac{1}{5}P_0$ or $\frac{4}{5}P_0$

So $\frac{4}{5}P_0 = P_0 (\frac{1}{2})^{3/h}$

solve for h...

$\frac{\frac{4}{5}P_0}{P_0} = \frac{P_0 (\frac{1}{2})^{3/h}}{P_0}$

$\frac{4}{5} = (\frac{1}{2})^{3/h}$

$\ln \frac{4}{5} = \ln (\frac{1}{2})^{3/h}$

$\ln \frac{4}{5} = \frac{3}{h} \ln \frac{1}{2}$

$\Rightarrow h = \frac{3 \ln \frac{1}{2}}{\ln \frac{4}{5}}$

13. Recall $[H^+]$ is the concentration of hydrogen ions in solution X measured in moles per liter (denoted M). Then pH level of solution $X = -\log[H^+]$. How many times more concentrated is $[H^+]$ of acid rain with a pH value of 3 to ordinary rain with a pH value of 6?

$[H^+]_a$ = concentration of acid rain

$[H^+]_n$ = concentration of normal rain

want $[H^+]_a = \, ? \, [H^+]_n$

or $\frac{[H^+]_a}{[H^+]_n} = \, ?$

So to find $[H^+]_a$

$3 = -\log [H^+]_a$

$-3 = \log [H^+]_a \Rightarrow [H^+]_a = 10^{-3}$

So to find $[H^+]_n$

$6 = -\log [H^+]_n$

$-6 = \log [H^+]_n \Rightarrow [H^+]_n = 10^{-6}$

$? = \frac{10^{-3}}{10^{-6}} = 10^3$

So 1000 times