Practice TMath 115 Final

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite different than those appearing here.

Let f & g, be functions with inverses f^{-1} and g^{-1} respectively.

T F $(x+3)^2 = x^2 + 9$

T F $(f \circ g)(x) = (g \circ f)(x)$

T F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$

T F $\sqrt{(x^2)} = x$ for all real numbers x.

T F If 2 is a root of g, then $g(2) = 0$.

T F $\ln \frac{x}{y} = \ln x - \ln y$ for all positive numbers x and y.

T F $\log(\log(10)) = 0$.

T F $f(f^{-1}(54)) = 54$

T F

T F

Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. Find all x such that
$$2(5 - (8 - x)^2)^{-\frac{1}{2}} - 1 = 0$$
.

2. Perform the opperation

$$\frac{\frac{2}{x^2} - x}{x - 2} + \frac{3x - 5}{(x + 4)(x - 4)} \qquad \frac{1 + 3i}{-6i + 2}$$

3. Given $m(x) = \dfrac{2x + 3}{x - 5}$, and $n(x) = \sqrt{4x - 8}$,

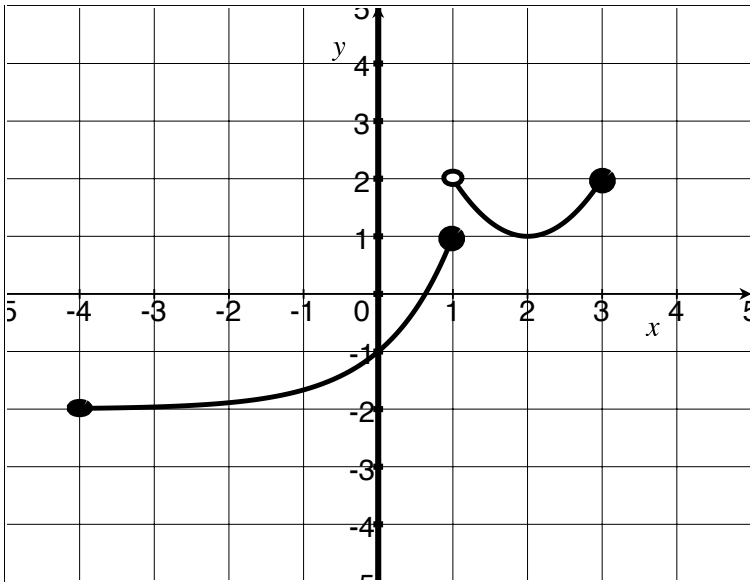
   (a) The inverse to the function m exists. Find m^{-1}.

   (b) If $p(x) = 3m(x + 1)$, find the domain and rule of p.

   (c) Find the domain and rule of $m \circ n$.

   (d) Find the domain and rule of $\dfrac{n}{m}$.

4. Let the following be the graph of g comprised of a parabola and an exponential function that have been shifted (not stretched).



(a) What is the domain of g?

(b) What is the range of g?

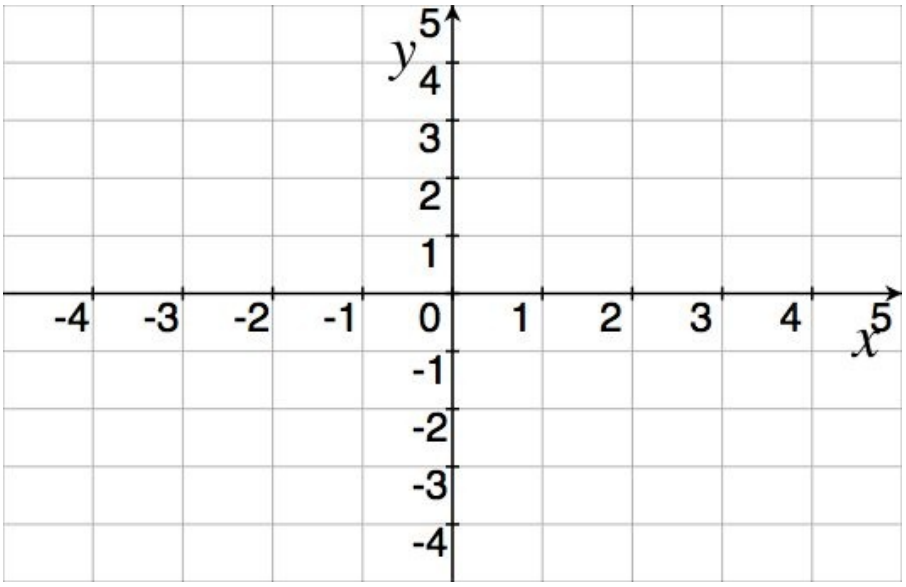
(c) Use the graph above to *estimate* all x value(s) so that $g(x) = 1$?

(d) Write down the piece-wise defined rule for g.

(e) Draw the graph of $-2g(x-1)$

5. Define f by

$$f(x) = \begin{cases} \frac{1}{x+2} & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2 \\ \log_2(x-1) & \text{if } 2 < x \leq 4 \end{cases}$$



(a) Graph f on the axes above.

(b) Find the following if possible:

$f(1)$

$\frac{4}{f(2)} + f(3)$

$f(0)$

$f(\frac{-1}{4})$

Domain of f

6. Find all of the exact values x that satisfy the following:

   $5^{5x}25^{x^2} = 125$ $\qquad$ $5^{4x-1} = 7^x$

7. Find all exact values for x that satisfy the following:

   $\log(x-16) = 2 - \log(x-1)$ $\qquad$ $\dfrac{15}{3+2\cdot 5^x} = 4$

8. Assume c, d, and z are all greater than zero and simplify:

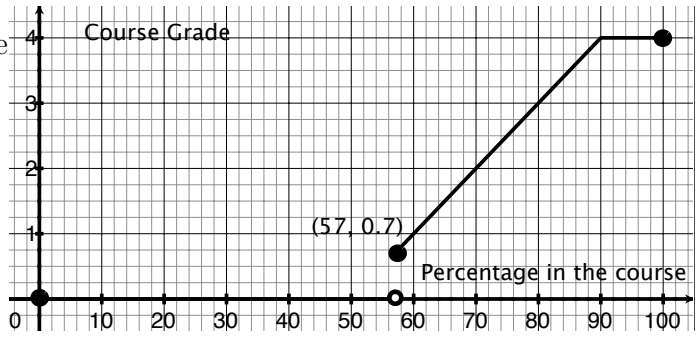
   $\dfrac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}}$ $\qquad$ $2 - \log_5(25z)$

9. Given $f(3) = 0$ find the other roots of $f(x) = x^4 - 3x^3 - 25x^2 + 75x$

10. Now that finals are next week, James T. Kirk would like to know if it is still possible to earn a 2.0. He has looked at the gradebook on MyMathLab and has computed the averages listed below.

    Find what grade he needs to get on the final to receive a 2.0 in the course. In case you don't remember, the weights specified in the syllabus and the graph of the function f that takes your class percentage x and returns your score on a 4. scale are also provided.

	weight	James' ave
Mini-Quizzes	5%	95%
WebAssign	10%	10%
WrittenHW	15%	0%
Quizzes	15%	70%
2 Exams	30%	100%
Final	25%	



11. A rancher with 180 meters of fencing intends to enclose a rectangular region along a river (which serves as a natural boundary requiring no fence).

    (a) Find the area of the region as a function of the width.

    (b) Find the maximum area that can be enclosed.

12. Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope

13. Recall $[H^+]$ is the concentration of hydrogen ions in solution X measured in moles per liter (denoted M). Then pH level of solution $X = -\log[H^+]$. How many times more concentrated is $[H^+]$ of acid rain with a pH value of 3 to ordinary rain with a pH value of 6?