notes:
(modify #2 so ax−h in the denominator
(Fix #3b
(Modify #4 x^2-3 $-1 \pm x$ $\frac{4}{x+2}$ $-4 \pm x$
(typo #9 & remove 9b

Exam 2

TMath 115

Practice

Note: This sample exam is to be used for practice. This is *not* a template for the exam that will be given in class. Many of the questions on the exam will look quite different than those here.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

   T F $\frac{3}{a} + \frac{4}{a^2} = \frac{7}{a+a^2}$ $\quad \frac{a}{a}\frac{3}{a} + \frac{4}{a^2} = \frac{3a+4}{a^2}$

   T F $x^2 + 8x + 15$ has a root at -5. $\quad (-5)^2 + 8(-5) + 15 = 25 - 40 + 15 = 0$

   T F $(x+2)^2 = x^2 + 4$ $\quad (x+2)^2 = (x+2)(x+2) = x^2 + 2x + 2x + 4$

   T F $\underset{f}{x^2 + 3}$ and $\underset{g}{\sqrt{x-3}}$ are inverses. consider -1: $\quad -1 \xrightarrow{f} (-1)^2 + 3 = 4 \xrightarrow{g} \sqrt{4-3} = +1$

   T F All functions have inverses.
   needs to pass the horizontal line test.

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Given that 2 is a root, find all the real and complex zeros of $x^4 + x^3 - 5x^2 - 2x$

2 is a root => $(x-2)$ is a factor

$(x-2) \cdot ? = x^4 + x^3 - 5x^2 - 2x$

$\Rightarrow ? = \frac{x^4 + x^3 - 5x^2 - 2x}{x-2}$

$$\begin{array}{r} x^3 + 3x^2 + x \quad R\ 0 \\ x-2 \,\big)\, x^4 + x^3 - 5x^2 - 2x \\ -(x^4 - 2x^3) \\ \hline 3x^3 - 5x^2 - 2x \\ -(3x^3 - 6x^2) \\ \hline x^2 - 2x \\ -(x^2 - 2x) \\ \hline 0 \end{array}$$

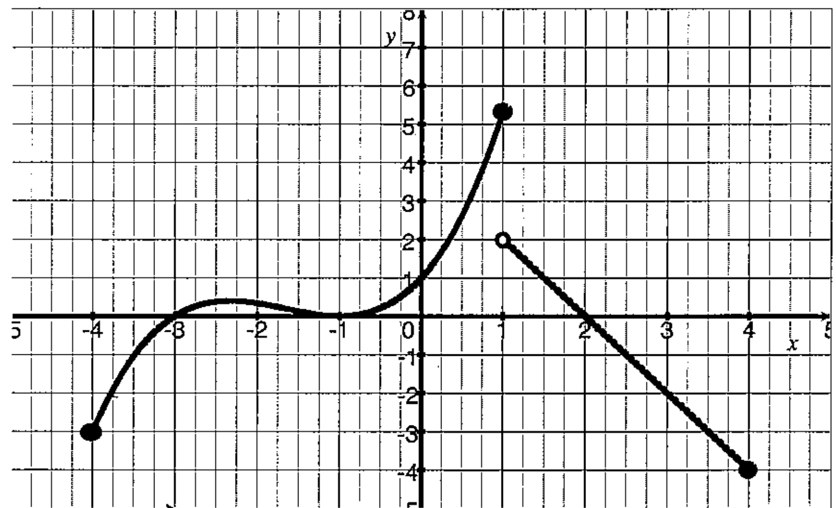
So
$(x-2)(x^3 + 3x^2 + x) = x^4 + x^3 - 5x^2 - 2x$
$(x-2)x(x^2 + 3x + 1) = x^4 + x^3 - 5x^2 - 2x$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$x = 2 \quad x = 0 \quad x = \frac{-3 \pm \sqrt{9 - 4(1)(1)}}{2(1)}$

$= \frac{-3 \pm \sqrt{5}}{2}$

3. Let f be the piece-wise defined function graphed below:



(a) Find the domain of f

$[-4, 4]$

or

$-4 \le x \le 4$

Estimate (b) ~~Find~~ the range of f

$[-3, 5.5]$

or $-3 \le y \le 5.5$

(c) Does f have an inverse? Why or why not?

No

fails the horizontal line test (see part e)

(d) Estimate the following *if* possible:

i. $f(1) \approx 5.5$

ii. $(f \circ f)(2) = f(f(2)) = f(0) = 1$

$2 \xrightarrow{f} 0 \xrightarrow{f} 1$

(e) Estimate all x such that $f(x) = -2$.

$x = 3$ and $x \approx -3.75$

(f) Identify the zeros of f.

$x = -3, -1$ and 2

(g) Assume f is comprised of a polynomial and a line. For the polynomial piece, assume when it is completely factored each real zero corresponds to a factor of the form $(x - c)^m$. Find the equation for f with least degree for the polynomial piece.

$$f(x) = \begin{cases} \text{polynomial} & -4 \le x \le 1 \\ \text{line} & 1 < x \le 4 \end{cases} = \begin{cases} \frac{1}{3}(x+3)(x+2)^2 & -4 \le x \le 1 \\ -2x + 4 & 1 < x \le 4 \end{cases}$$

Polynomial

-3 is a root $\Rightarrow (x - -3)$ is a factor

-1 is a root $\Rightarrow (x - -1)$ is a factor

@ $x = -3$ looks like line $\Rightarrow (x - -3)^1$ is a factor

@ $x = -1$ looks like parabola $\Rightarrow (x - -1)^2$ is a factor

passes thru $(0,1)$ so

$1 = a(0+3)(0+1)^2$

$1 = a \cdot 3$

$\frac{1}{3} = a$

line

slope $= -2$

thru $(2,0)$

so

$0 = -2(2) + b$

$\Rightarrow b = 4$

4. Let $h(x) = \begin{cases} \frac{4}{x+2} & -3 < x \leq -1 \\ x^2 - 3 & -1 < x \leq 4 \end{cases}$

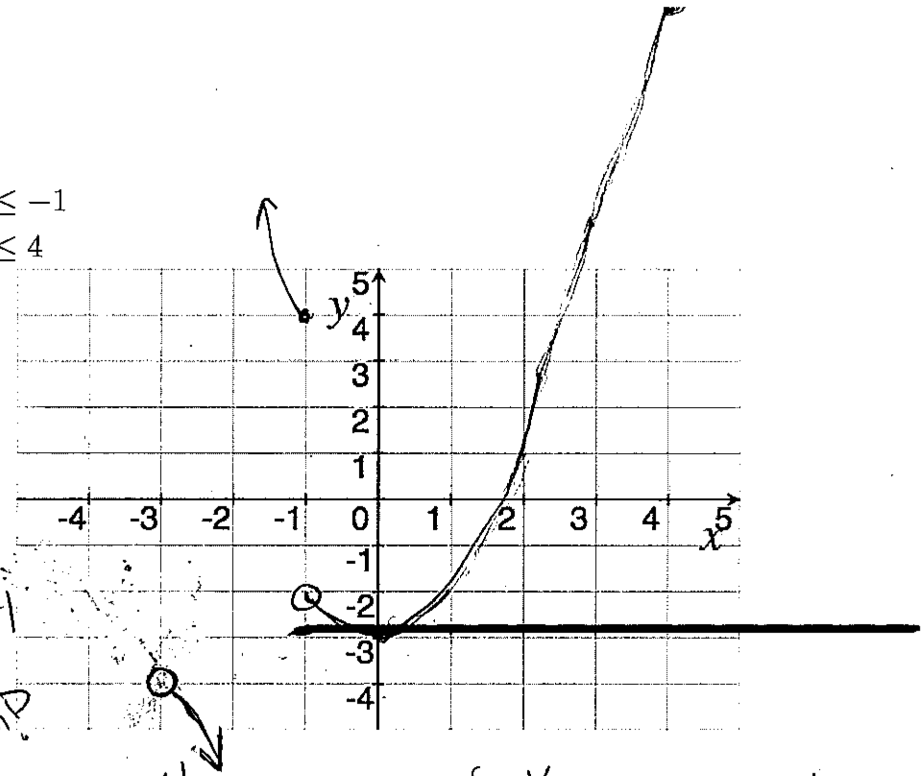
(a) Estimate the following *if* possible:

i. $h(0)$ $-1 < 0 \leq 4$

so $0^2 - 3 = -3$

ii. $(h)(-2)$ $-3 < -2 \leq -1$

$\frac{4}{-2+2} = \frac{4}{0}$ no solution?



(b) Graph h.

$x^2 - 3$

parabola shifted down 3 units

$\frac{4}{x+2}$

graph of $1/x$ vertically stretched by 4 shifted left 2 units

5. Consider the function $r(x) = \dfrac{-2x-1}{x+2}$.

(a) Find the domain of r.

denominators can't equal zero

$x+2 \neq 0$

$x \neq -2$ or $(-\infty, -2) \cup (-2, \infty)$

(b) Given that r is one-to-one (i.e. r has an inverse, find f^{-1}.

$y = \dfrac{-2x-1}{x+2}$

swap x's + y's

$x = \dfrac{-2y-1}{y+2}$

now solve for y...

$y+2 \; x = \dfrac{-2y-1}{\cancel{y+2}} \; \cancel{y+2}$

$x(y+2) = -2y-1$

$xy + 2x = -2y - 1$

$+2y \qquad +2y$

$xy + 2y + 2x = -1$

$-2x \qquad -2x$

$xy + 2y = -1 - 2x$

$y(x+2) = -1 - 2x$

$\dfrac{y\cancel{(x+2)}}{\cancel{x+2}} = \dfrac{-1-2x}{x+2}$

$y = \dfrac{-1-2x}{x+2}$

6. Let $p(x) = 2x^2 + 10x - 5$

$a(x-h)^2+k$

(a) Write $p(x)$ in vertex or standard form.

$\frac{y}{2} = \frac{2x^2+10x-5}{2}$

$\frac{1}{2}y = x^2+5x-\frac{5}{2}$

$+(\frac{5}{2})^2 \quad +(\frac{5}{2})^2$

$\frac{1}{2}y + \frac{25}{4} = \underbrace{x^2+5x+\frac{25}{4}} - \frac{5}{2}$

$\frac{1}{2}y + \frac{25}{4} = (x+\frac{5}{2})^2 - \frac{5}{2}$

$\frac{1}{2}y + \frac{25}{4} = (x+\frac{5}{2})^2 - \frac{5}{2}$
$\quad -\frac{25}{4} \qquad\qquad -\frac{25}{4}$

$\frac{1}{2}y = (x+\frac{5}{2})^2 - \frac{35}{4}$

$\times 2 \qquad \times 2$

$y = 2(x+\frac{5}{2})^2 - 2\cdot\frac{35}{4}$

$y = 2(x+\frac{5}{2})^2 - \frac{35}{2}$

or

x-coord of vertex $= h = -b/2a$
$= -10/2\cdot 2 = -5/2$

y-coord of vertex $= K = p(-5/2)$
$= 2(-\frac{5}{2})^2 + 10(-\frac{5}{2}) - 5$
$= \frac{+25}{2} - \frac{50}{2} - \frac{10}{2}$
$= -\frac{25}{2} - \frac{10}{2} = -\frac{35}{2}$

So $2(x+\frac{5}{2})^2 - \frac{35}{2}$

(b) Identify the graph transformations used to transform the graph of $y = x^2$ into p.

$2(x+\frac{5}{2})^2 - \frac{35}{2}$

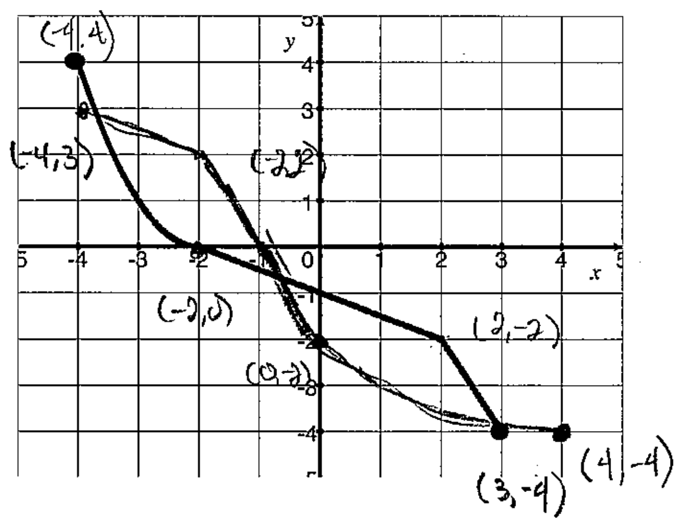
1) vertical stretch by factor of 2 / multiply y-coord by 2

2) vertical shift down $\frac{35}{2}$ units

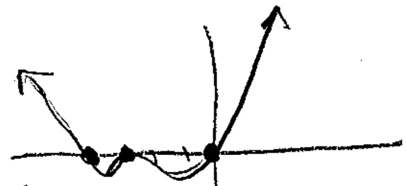
3) horizontal shift left $\frac{5}{2}$ units

7. Consider the graph of g. Graph g^{-1}

Recall if (x, y) is on the graph of g then (y, x) is on the graph of g^{-1}.



8. Sketch the graph of $\frac{1}{2}x(x+3)^2(x+4)$.



4th deg polynomial => end behavior is the same

Positive leading coefficient ($\frac{1}{2}$) => as $x \to \infty$, $y \to \infty$

and as $x \to -\infty$, $y \to \infty$

x is a factor => 0 is a root

$x+3$ is a factor => -3 is a root

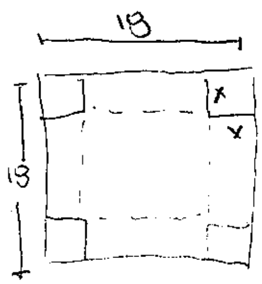
$x+4$ is a factor => 4 is a root

$(x+3)^2$ is a factor => touches x-axis + does not cross

rule: must be smooth with no corners or breaks

9. A square piece of a tin 18 inches on each side is to be make into a box, without a top, but cutting a square from each corner and folding up the flaps to corm the sides.

(a) Find the volume v of the box as a function of the length of the squares removed from the corners.



$V = \text{length} \cdot \text{width} \cdot \text{height}$

$= (18-2x)(18-2x)(x)$

(b) What size corners should be cut so that the volume of the box is 432 cubic inches?

i.e. find x so $v = 432$

$432 = (18-2x)(18-2x)(x)$

need to solve a cubic....

at this point of the class we'd need the graph to read it?