TMath 115

Note: This sample exam is to be used for practice. This is *not* a template for the exam that will be given in class. Many of the questions on the exam will look quite different than those here.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

   T F $\frac{3}{a} + \frac{4}{a^2} = \frac{7}{a+a^2}$

   T F $x^2 + 8x + 15$ has a root at -5.

   T F $(x+2)^2 = x^2 + 4$

   T F $x^2 + 3$ and $\sqrt{x-3}$ are inverses.

   T F   All functions have inverses.

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Given that 2 is a root, find all the real and complex zeros of $x^4 + x^3 - 5x^2 - 2x$

3. Let f be the piece-wise defined function graphed below:

(a) Find the domain of f

(b) Find the range of f


(c) Does f have an inverse? Why or why not?
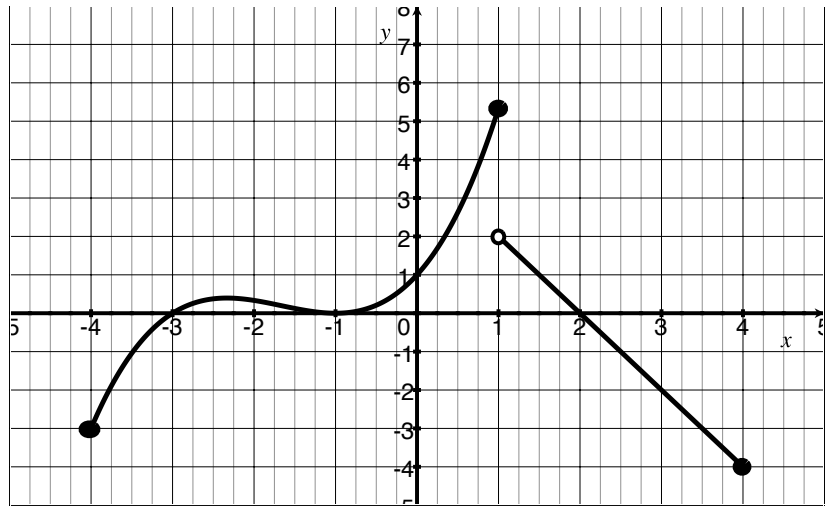
(d) Estimate the following *if* possible:

   i. $f(1)$

   ii. $(f \circ f)(2)$

(e) Estimate all x such that $f(x) = -2$.

(f) Identify the zeros of f.

(g) Assume f is comprised of a polynomial and a line. For the polynomial piece, assume when it is completely factored each real zero corresponds to a factor of the form $(x - c)^m$. Find the equation for f with least degree for the polynomial piece.

4. Let $h(x) = \begin{cases} \frac{4}{x+2} & -3 < x \le -1 \\ x^2 - 3 & -1 < x \le 4 \end{cases}$

(a) Estimate the following *if* possible:

i. $h(0)$

ii. $(h)(-2)$
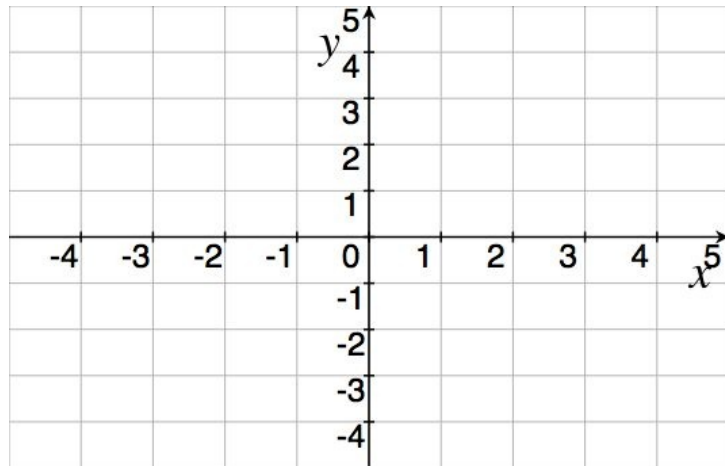
(b) Graph h.

5. Consider the function $r(x) = \dfrac{-2x - 1}{x + 2}$.
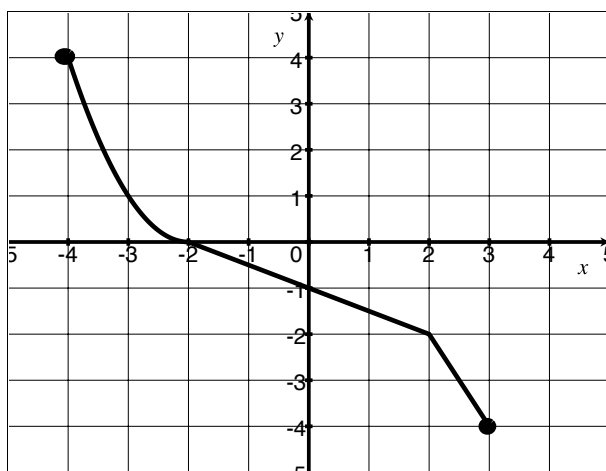
(a) Find the domain of r.

(b) Given that r is one-to-one (i.e. r has an inverse, find f^{-1}.

6. Let $p(x) = 2x^2 + 10x - 5$

   (a) Write $p(x)$ in vertex or standard form.

   (b) Identify the graph transformations used to transform the graph of $y = x^2$ into p.

7. Consider the graph of g. Graph g^{-1}

8. Sketch the graph of $\frac{1}{2}x(x+3)^2(x+4)$.

9. A square piece of a tin 18 inches on each side is to be make into a box, without a top, but cutting a square from each corner and folding up the flaps to corm the sides.

   (a) Find the volume v of the box as a function of the length of the squares removed from the corners.

   (b) What size corners should be cut so that the volume of the box is 432 cubic inches?