Polynomials

A *polynomial function of degree n* is a function of the form

$$f(x) = a_n x^x + a_{n-1}x^{n-1} + ...a_2x^2 + a_1x + a_0$$

where n is a nonnegative integer and the coefficients a_n, a_{n-1}, $...a_2$, a_1, a_0 are real numbers with $a_n \neq 0$. The term $a_n x^n$ is called the *leading term*, the number a_n is called the *leading coefficient*, and a_0 is the *constant term*.

1. For each of the expressions below, determine if it is a polynomial, and *if it is*, determine the degree:

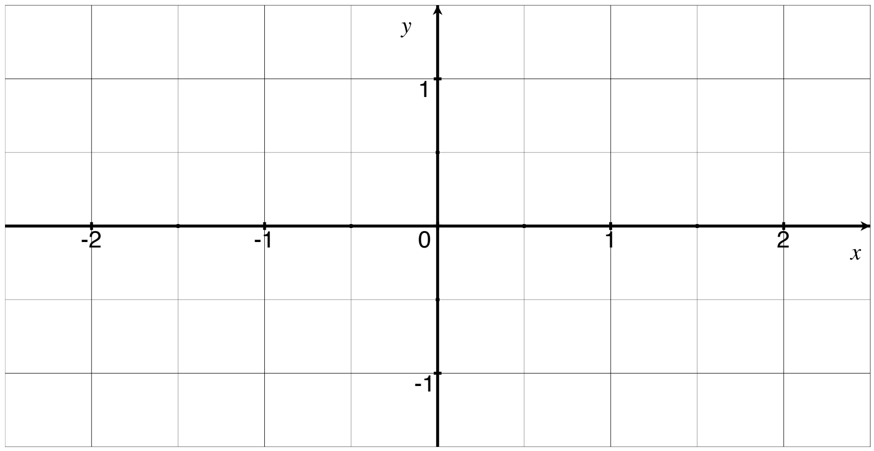
expression	polynomial? (yes/no)	leading term (if applicable)	degree (if applicable)
$117x^4 + 6x^{12} + x$			
$2^x - 5x^2$			
$\sqrt{5}x^2 - \pi = (\sqrt{5})x^2 - \pi$			
$7x^8 - 4.56x^4 - 7x^8 + x^2$			
3			
0			

We have already spent a few days on first degree and second degree polynomials (i.e. lines & parabolas). We now turn to higher order polynomials.

2. Fill out as much of the following table as necessary to sketch a graph of the function

$$f(x) = x^3.$$

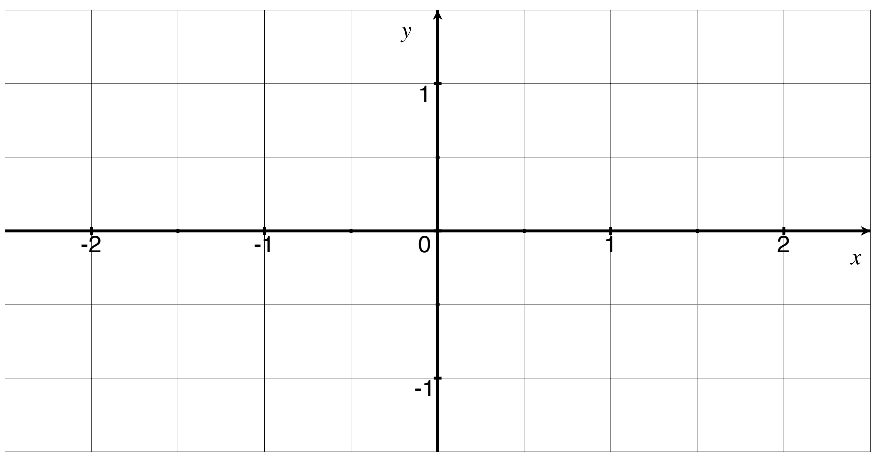
x	$f(x) = x^3$
-1	
$\frac{-1}{4}$	
0	
$\frac{2}{3}$	



3. Fill out as much of the following table as necessary to sketch a graph of the function

$$g(x) = x^5.$$

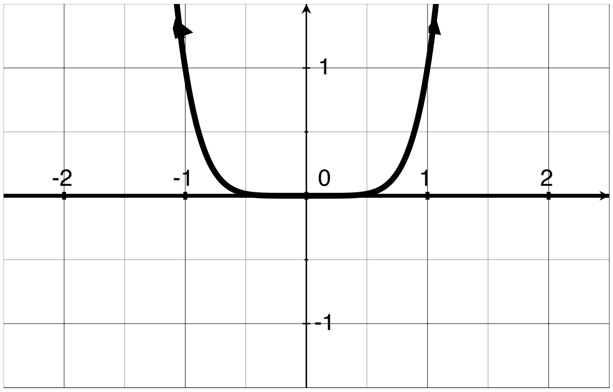
x	$g(x) = x^5$
$\frac{-1}{2}$	
0	
$\frac{1}{3}$	
1	

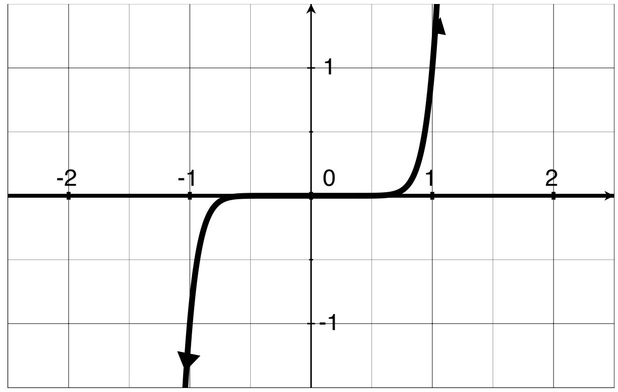


4. Do you see any similarities between the graphs of x^3 and x^5? What do you think the graph of $h(x) = x^7$ would look like?

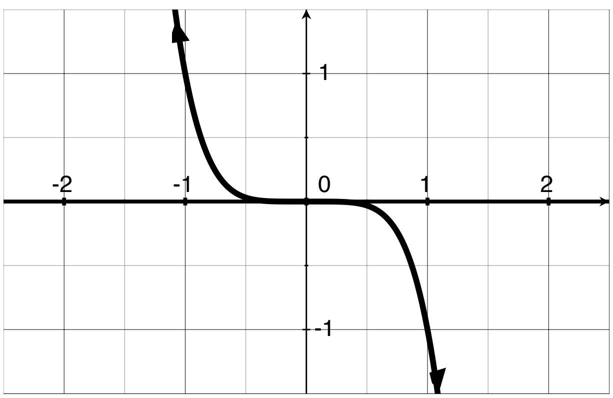
5. Consider the "end behavior" of polynomials like x^3, x^5, and x^7. That is, what happens when x "gets very large in the positive direction" ($x \to \infty$) and what happens when x "gets very large in the negative direction" ($x \to -\infty$)?

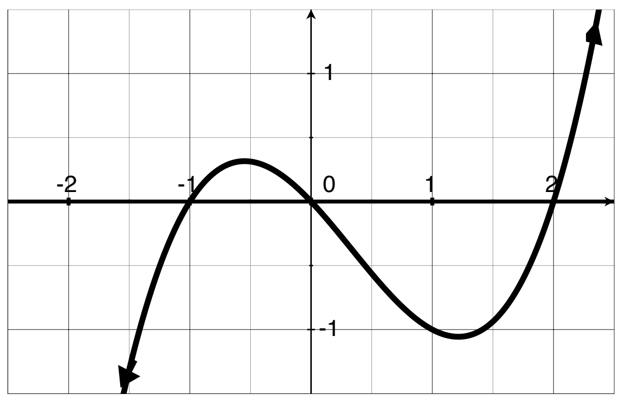
6. Which of the following could be a graph of x^{11}? Why or why not?





7. Use your observations of "end behavior" and identify which (if any) of the following could be the graph of an odd degree polynomial? (Not necessarily of the form x^n but like the polynomials you looked at on page 1)?





8. Let $f(x) = x^3$. Use question 5 and §1.5 (what was that over again?) to graph:

$m(x) = -f(x+1) = -(x+1)^3$ and $n(x) = -f(x) - 1 = -x^3 - 1$