Inverses

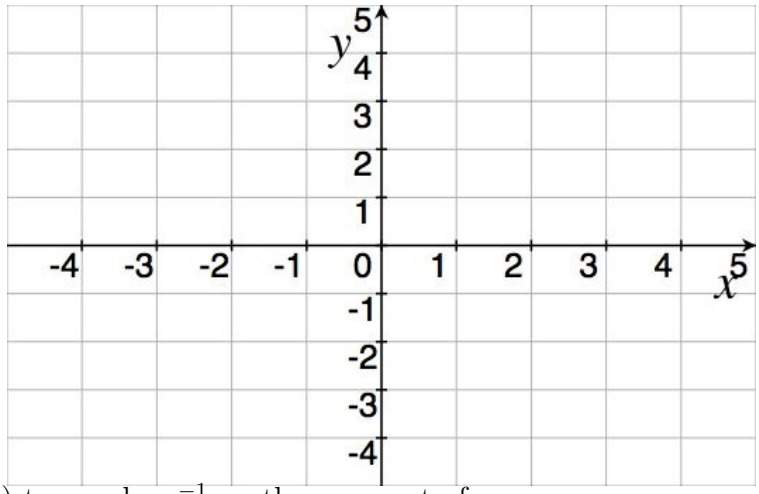
1. Given a tube partway filled with liquid will have a height dependent on the temperature. That is, we have height h as a function of Temperature T.

   (a) What does $h(32) = 1$ mean in physical terms?

   (b) Describe the inverse function h^{-1} by identifying the inputs, outputs, and what it measures.

2. Let m be the function completely defined by the table:

$\star$	$m(\star)$	$\star$	$m^{-1}(\star)$
1	-3	-3	
$\frac{3}{2}$	2	2	
π	$\sqrt{2}$	$\sqrt{2}$	



   (a) Complete the table above to define m^{-1}.

   (b) Plot the graph of m on the set of axes provided.

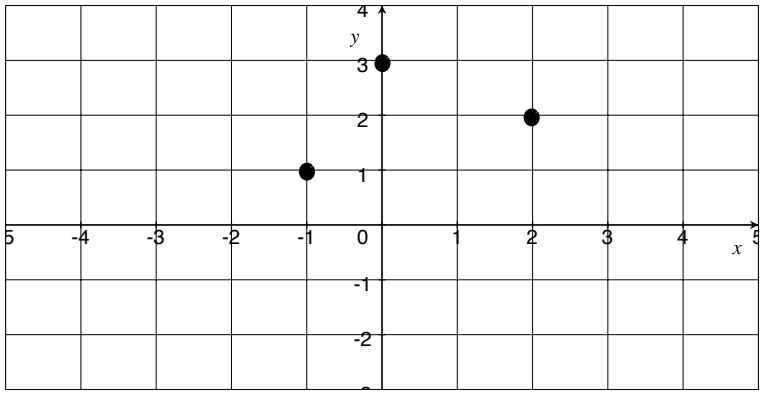
   (c) Use a different mark (or color) to graph m^{-1} on the same set of axes.

   (d) Notice the point (1,-3) is on the graph of m and (-3,1) is on the graph of m^{-1}. Similarly $(\frac{3}{2}, 2)$ is on the graph of m and $(2, \frac{3}{2})$ is on the graph of m^{-1}.

   (e) Find the domain of m and range of m^{-1}. Are there any similarities?

The observations you made in (e) is true in general, and more:
if f is the inverse of g then: Domain of f=Range of g Range of f=Domain of g

3. Let n be the function defined by the following graph:



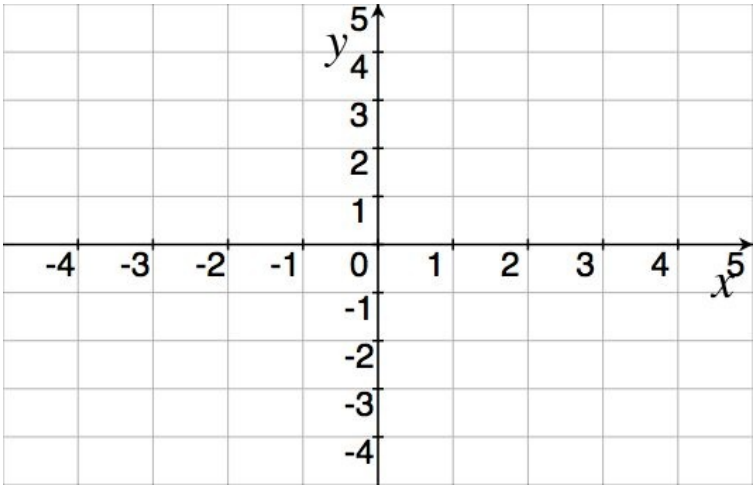
   (a) Will n have an inverse? Why?

   (b) Use the observations from #1d to graph n^{-1}.

4. Let p be the function defined by $p(x) = x^2 - 1$.

(a) Draw the graph of p.

(b) Will p have an inverse? Why?



(c) Let the function q have the same rule as p (so $q(x) = x^2 - 1$), but with a *restricted* domain. The domain of q is set to all $x \geq 0$ (in interval notation: $[0, \infty)$). Draw the graph of q with distinct marks from the graph of p.

(d) Will q have an inverse? Why

(e) Sketch the graph of q^{-1} on the above set of axes. Verify your answer by looking at example 8 on page 101.

When we are given a function that is *not* one-to-one we can choose to restrict the domain to a subsection and in so doing, define a partial inverse.