Exponent Properties

There are 'five' properties of exponents that you need to know well to work with algebraic expressions and equations. The 'zeroth' property is:

Property 0) Let b be a real number and m a positive integer, then

$$b^m = b \cdot b \cdot b \cdot \ldots \cdot b$$

where b appears m times on the right.

For example, we can write out $(-2)^4$ with no exponents as $(-2) \cdot (-2) \cdot (-2) \cdot (-2)$ or $(4)^3$ as $4 \cdot 4 \cdot 4$. We will often rely on the order of operations and drop parenthesis and just write 4^3 instead.

Warning: the order of operations is such that exponents *only* effect the object they 'hover' behind. For example, $-2^4 = -2 \cdot 2 \cdot 2 \cdot 2$, since the exponent is only effecting the 2 and not the negative sign.

There are some special cases of the above definition (that not all mathematicians agree on). Mainly:

$$0^m = 0 \qquad 1^m = 1 \qquad b^0 = 1 \qquad 0^0 \text{ is undefined}$$

1. Consider $x^2 \cdot x^5$.
   (a) Write out $x^2 \cdot x^5$ with no exponents (like what was done in the above paragraph).
   (b) Write $x^2 \cdot x^5$ as x raised to a single power.
2. Consider $y^3 \cdot y^3$.
   (a) Write out $y^3 \cdot y^3$with no exponents.
   (b) Write $y^3 \cdot y^3$ as y raised to a single power.
3. Use your work above the finish the following first property:

Property 1) Let b be a real number, and let m and n be positive integers, then

$$b^m \cdot b^n =$$

1. Write out $\square^4$ with no exponents (like you did on the first page).

2. Write an $(x^3)^4$ using only multiplication and instances of x^3. In other words, do the same problem as number one but instead of using a square, use the symbol x^3.

3. Now expand the above with Property 0 to write out $(x^3)^4$ with no exponents.

4. Repeat the steps above to write out $(y^5)^2$ with no exponents.

5. Use your work above to finish the following:

Property 2) Let b be a real number, and let m and n be positive integers, then

$$(b^m)^n =$$

1. Let x and y be real numbers and m be positive integers. Is there a relationship between $(x \cdot y)^m$ and $x^m \cdot y^m$? If so, state it and explain why it is true.

By answering the previous question you can now state:

Property 3) Let a and b be real numbers, and let m be a positive integer, then

$$(a \cdot b)^m =$$

You can verify these properties on page 14 of your text. I suggest you *do not* memorize them as most people mix them up. Rather, work through examples like 2 on page 1 and 4 on page 2 whenever you realize you need to work with exponents.

The remaining two properties are:

Property 4) Let b be a real number and let m and n be positive integers, then

$$\frac{b^m}{b^n} = b^{m-n}.$$

and

Property 5) Let a and b be a real numbers and let m be a positive integer, then

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m.$$

1. Recall that $x^{-2} = \frac{1}{x^2}$. Make sure Property 4 is consistent with this observation by simplifying

$$\frac{x^6}{x^8}$$

All the rules can be found in Appendix 1 on pages 788 & 189. Now, some practice...

1. Simplify $(-4x^2y^3)(7x^3y)$.

2. Simplify $\left(\dfrac{x^5}{2y^{-3}}\right)^{-3}$. Hint: use *lots* of room and make sure your exponents don't fall!

3. Evaluate the expression and write the result in the form $a + bi$: $(-5 + 2i)^3$

4. Solve for x given $3x^{-2} - 7 = 0$

Notice that you can verify your answers to 1-2 by looking at Example 7 on page 790. Number four comes up a great deal in calculus word problems.