Exam 2

TMath 115

45

Autumn 2015

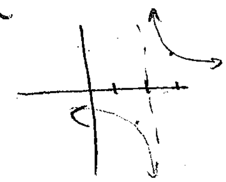
NAME: Key

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function with an inverse defined.

   T F $\dfrac{\frac{1}{2}+\frac{1}{3}}{a} = \dfrac{5}{6a}$ $\qquad \dfrac{\frac{3}{3}\frac{1}{2}+\frac{1}{3}\frac{2}{2}}{a} = \dfrac{\frac{3+2}{6}}{a} = \dfrac{5/6}{a}\cancel{\tfrac{6}{6}} = \dfrac{5}{6a}$

   T F The graph of $\frac{1}{x-2} = y$ has a vertical asymptote at $x = 2$.

   → graph of $1/x$ shifted right 2 units

   T F $(x+2)^2 = x^2 + 4$ $\qquad (x+2)^2 = (x+2)(x+2) = x^2+2x+2x+4$

   T F $(x+1)$ is a factor of $x^4 - 3x^2 + 2$. $\quad x+1$ is a factor $\Leftrightarrow -1$ is a root

   $(-1)^4 - 3(-1)^2 + 2 = 1 - 3 + 2 = 0$

   T F $(f \circ f^{-1})(55) = 55$

   T F All functions have inverses. consider $y = x^2$ both 2 and -2 go to 4

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Let $q(x) = -2x^2 + 12x - 25$.

   $a(x-h)^2+k=y$ (+.5)

   (a) [3] (PracticeExam #6) Write q in vertex or standard form. (+.5)

   (+.5) $\dfrac{y}{-2} = \dfrac{-2x^2+12x-25}{-2}$

   $-\frac{1}{2}y = x^2 - 6x + \frac{25}{2}$

   (+.5) $+\left(\frac{-6}{2}\right)^2 \qquad +\left(\frac{-6}{2}\right)^2$

   $-\frac{1}{2}y + 9 = (x-3)^2 + \frac{25}{2}$ (+.5)

   $-9 \qquad -9$

   $-\frac{1}{2}y = (x-3)^2 + \frac{7}{2}$ alg (+1) or

   $y = -2(x-3)^2 - 7$

   x-coord $= -b/2a = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3 = h$

   y-coord $= -2(3)^2 + 12(3) - 25$ (+.5)

   $= -18 + 36 - 25 = -7 = k$

   So

   $y = -2(x-3)^2 - 7$ form (+.5)

   (+.5) (+.5) (+.5)

   (b) [2] (WebHW8 #4) Identify if the vertex is a minimum or a ~~maximum~~ and justify your answer.

   The vertex is a max (+1)

   (+1) b/c the -2 flipped the parabola vertically so the graph looks like:

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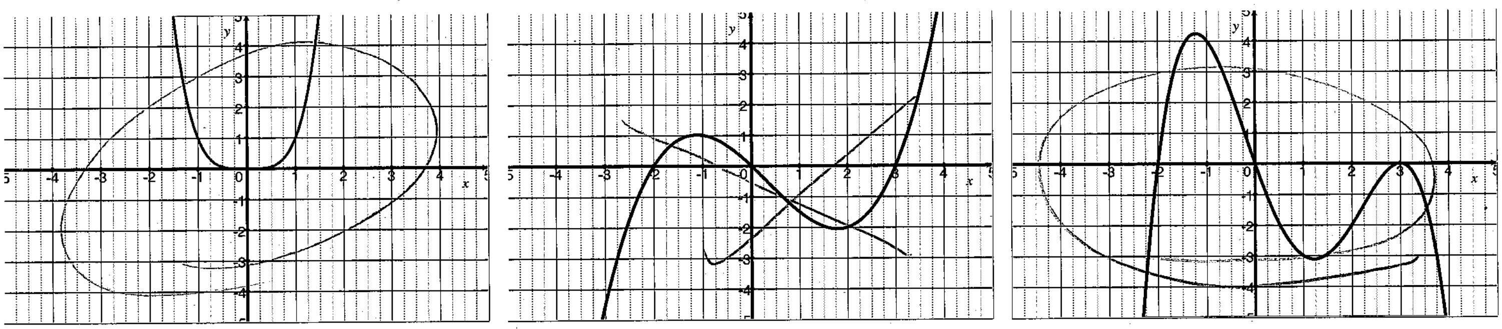
+.5 +.5 +.5

3. [2] (WebHW10 #3) Write a polynomial of degree five that has five distinct x-intercepts and whose graph rises to the left and falls to the right.

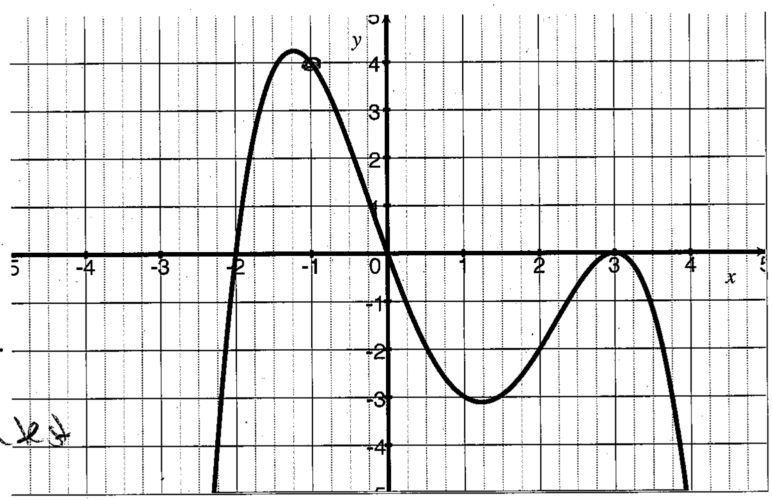
+.5

lots of answers $-8x(x+1)(x+2)(x+3)(x+4)$ or $\frac{-1}{3}(x+50)(x-2)(x+2)(x-7)(x-8)$

4. [2] (Quiz3 #2) Identify all of the graphs below that could be a 4th degree polynomial.



5. Let p be the function graphed below.



(a) [1] (PracticeExam #3)
~~Find~~ the range of p.
Estimate

y-values (+.5)
got it (+.5)

$(-\infty, 4.3]$

or $y \leq 4.3$

(b) [1] (PraticeExam #3)
True or False
The function p has an inverse.

False (+1)
fails the horizontal line test

(c) [1] (Quiz3 #4) True or False
The leading coefficient of p is negative.

True (+1)

(d) [4] (§2.3 #38) Assume when p is completely factored, each real zero corresponds to a factor of the form $(x-c)^m$. Find the equation of least degree for p.

(+1)
-2 is a root $\Rightarrow x - -2$ is a factor
0 is a root $\Rightarrow x - 0$ is a factor
3 is a root $\Rightarrow x - 3$ is a factor

(+1)
@ $x = -2$ looks like a line $\Rightarrow (x - -2)$ is a factor
@ $x = 0$ looks like a line $\Rightarrow (x)$ is a factor
@ $x = 3$ looks like a parabola $\Rightarrow (x-3)^2$ is a factor

So $a(x+2)(x)(x-3)^2 = y$

read a point (+.5) passes thru $(-1, 4)$ so

plug in (+.5) $a(-1+2)(-1)(-1-3)^2 = 4$

$\Rightarrow a(1)(-1)(-4)^2 = 4$

$a \cdot -16 = 4$ alg (+.5)

$\Rightarrow a = -1/4$

got it (+.5) So $y = -1/4(x+2)x(x-3)^2$

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6. Let $f(x) = \dfrac{x+1}{x-2}$

(a) [3] (WebHW11 #4) Find the quotient and remainder and consider writing your answer as Quotient + Remainder/$(x-2)$

$(+.5)$ 1 R3 $(+.5)$

$$x-2 \overline{)\, x+1}$$
$$-(x-2)$$
$$3$$

Note $f(x) = \dfrac{x+1}{x-2} = 1 + \dfrac{3}{x-2}$

?? which looks like #7 ??

set up (+.5) arithmetic (+.5)
algorithm (+1)

(b) [1] (RationalWks #2)
Find the domain of f

all x but 2 or $x \neq 2$ or $(-\infty, 2) \cup (2, \infty)$

(c) [4] (§1.7 #55) Find the inverse of f

(+1) $x = \dfrac{y+1}{y-2}$

clear den (+.5)
algebra (+1.5)

y on one side (+.5)
only one y on one side (+.5)

$x(y-2) = y+1$
$xy - 2x = y + 1$
$\quad -y \quad\quad -y$
$xy - y - 2x = 1$
$\quad\quad +2x \quad +2x$

$xy - y = 1 + 2x$
$\dfrac{y(\cancel{x-1})}{\cancel{x-1}} = \dfrac{1+2x}{x-1}$

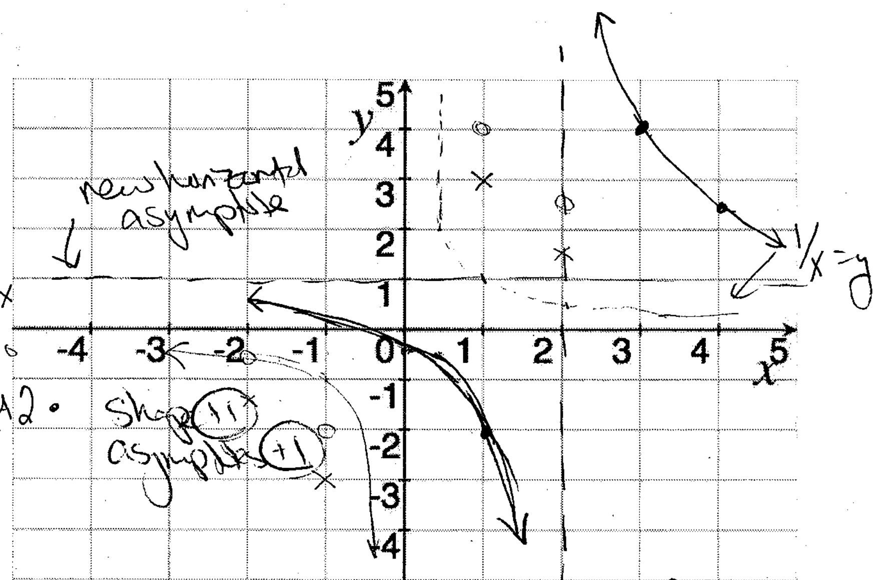
$y = \dfrac{1+2x}{x-1}$

7. Consider $g(x) = 1 + \dfrac{3}{x-2}$

$1 + \frac{3}{2} \cdot 1 - \frac{3}{2}$

(a) [4] (RationalWks #4)
Graph g.
(Consider using graph transformations of $y = \frac{1}{x}$!)

$1 + \dfrac{3}{x-2}$
(+.5) vertical stretch by 3 x
(+.5) vertical shift up by 1 ∘
(+.5) horizontal shift right 2 •
(+.5) order



(b) [1] (WebHW12 #5)
Find any vertical asymptotes.

$x = 2$

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22
23
45

8. [3] (§2.3 #65) The area of a rectangle is $(2x^4 - 2x^3 + 5x^2 - x + 2)$ square centimeters. Its length is $(x^2 - x + 2)$ cm. Find its width.

area = length · width (+.5)

$\Rightarrow$ width = $\frac{\text{area}}{\text{length}}$

$= \frac{2x^4 - 2x^3 + 5x^2 - x + 2}{x^2 - x + 2}$ (+.5)

$= 2x^2 + 1$ (+.5) (+.5)

$$\begin{array}{r} 2x^2 \quad\quad\quad +1 \\ x^2-x+2 \,\big)\, \cancel{2x^4} - \cancel{2x^3} + 5x^2 - x + 2 \\ -(\cancel{2x^4} - \cancel{2x^3} + 4x^2) \\ \hline x^2 - x + 2 \end{array}$$

algorithm (+1)

9. [2] (ModelingWks #1) Alisha went to Europe last summer. She discovered that when she exchanged her U.S. dollars for euros, she received 25% fewer euros than the number of dollars she exchanged. When she returned to the United States, she got 25% more dollars than the number of euros she exchanged.
Are the two conversion functions inverses? Justify your reasoning.

No? (+.5) reasoning (+1) sense/clear (+.5)

Let d be # of US dollars & e be # of euros.
Then $e(d) = d - .25d = .75d$
converts dollars to euros
and $d(e) = e + .25e = 1.25e$
converts euros to dollars
Notice $(d \circ e)(d) = d(e(d)) = d(.75d) = 1.25 \cdot .75d \neq d$

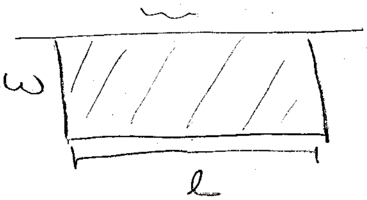
If Alisha started w/ $100 and the 2 conversion functions were inverses she should be able to convert her $ & get $100 back but
$100 → 75 euros → $93.7
She doesn't

10. (WebHW8 #6) A rancher with 180 meters of fencing intends to enclose a rectangular region along a river (which serves as a natural boundary requiring no fence).

(a) [3] Find the area of the region as a function of the width.

Area = $l \cdot w$ (+.5)

start (+.5) width = w
identify variables (+.5)

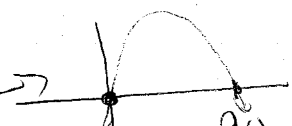


note $l + w + w = 180$
$\Rightarrow l + 2w = 180$ (+1)
$\Rightarrow l = 180 - 2w$

So Area = $(180 - 2w) \cdot w$ (+.5)

(b) [2] Find the maximum area that can be enclosed.

Area $= (180 - 2w)w = 180w - 2w^2 = -2w^2 + 180w$
So parabola opening down.
(+.5) The max happens at the vertex?



report Area (+.5)

(+1) x-coord $= -b/2a = \frac{-180}{2(-2)} = 45$

$y = -2w^2 + 180w$
or $-\frac{1}{2}y = w^2 - 90w$
$+45^2 = (w-45)^2 \Rightarrow y = -2(w-45)^2 + 40$

between 0 and 90 or 45
⇒ Area is (plug in 45)

So max area is $-2(45)^2 + 180(45) = 4050$

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