Exam 1 **TMath 115** Autumn 2015

NAME: Key

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T (F) $\frac{3}{a} + \frac{4}{a^2} = \frac{3}{a} + \frac{\sqrt{4}}{\sqrt{a^2}} = \frac{5}{a}$ $\quad \frac{a}{a}\frac{3}{a} + \frac{4}{a^2} = \frac{3a+4}{a^2}$

(T) F $x^2 + 8x + 15 = (x+3)(x+5)$ $\quad x^2+5x+3x+15 = x^2+8x+15$

T (F) $(x+2)^2 = x^2 + 4$ $\quad (x+2)(x+2) = x^2+2x+2x+4 = x^2+4x+4$

T (F) The domain of $a(x) = \sqrt{x-4}$ is $(4, \infty)$ $\quad x-4 \geq 0$, $x \geq 4$ so $[4, \infty)$

T (F) $\sqrt{i} = -1$ $\quad \sqrt{-1} = i$ so $i^2 = -1$

T (F) If 1kg=2.2lbs, then 4kg equals 1.8lbs $\quad 4\text{kg} \cdot \frac{2.2\text{lbs}}{1\text{kg}} = 9.84\text{lbs}$

T (F) $f(x-1) = f(x) - 1$ $\quad$ let $f(x) = x^2$

$f(x-1) = (x-1)^2 \neq x^2 - 1 = f(x) - 1$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] (PracticeExamWks #1) Find $\dfrac{\frac{2}{x^2} - x}{x-2} + \dfrac{3x-5}{(x+4)(x-4)}$

$x^2 \quad \dfrac{\frac{2}{x^2} - x}{x-2} + \dfrac{3x-5}{(x+4)(x-4)}$

$\dfrac{2x^{\not 2}}{x^{\not 2}} - x^2 x$ $\quad$ $x^2(x-2)$ $\quad + \dfrac{3x-5}{(x+4)(x-4)}$

$\dfrac{2-x^3}{x^2(x-2)} + \dfrac{3x-5}{(x+4)(x-4)}$

$\dfrac{(x+4)(x-4)(2-x^3)}{(x+4)(x-4)x^2(x-2)} + \dfrac{3x-5}{(x+4)(x-4)} \dfrac{x^2(x-2)}{x^2(x-2)}$

$\dfrac{(x+4)(x-4)(2-x^3) + (3x-5)x^2(x-2)}{(x+4)(x-4)x^2(x-2)}$

stated (+1.5)

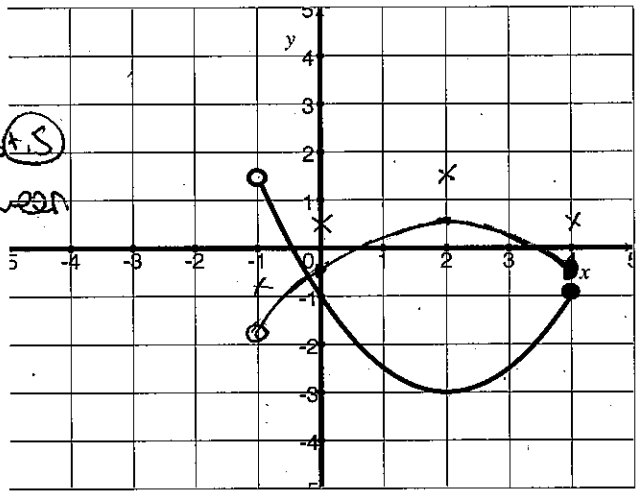
fraction in fraction (+1)

common den (+1.5)

added fractions (+1.5)

arithmetic/notation (+1.5) 10

3. Let f be the parabola with a restricted domain that is shown below:



(a) [2] (WebHW3 #19)
Find the domain of f
$(-1, 4]$ brackets +.5
+.5 +.5
+.5 the x-values go between
$-1 < x \leq 4$

(b) [4] (WebHW3 #16 & WebHW6 #17)
Estimate the following *if* possible:

i. $f(3) \approx -2.5$ +1

ii. $(f \circ f)\left(\frac{1}{2}\right) = f(f(\frac{1}{2})) \approx f(2)$ not defined? trick question?
composition +.5 +.5 +.5

iii. $3f(2) = 3(-3) = -9$ graph reading +.5
+.5 +.5

(c) [2] (GraphTransformation2 #2) Estimate x so that $f(x) = -2$.
find inpts so outputs are -2 +.5
so $\approx \frac{1}{2}$ and 3.5 got one +.5
+.5 +.5

(d) [3] (Quiz2 #3) Draw the graph of g if $g(x) = -\frac{1}{2}f(x) - 1$.
order +.5 +.5 1) vert flip + stretch by 1/2 ×××
got it +1.5 +.5 2) vert shift down by 1

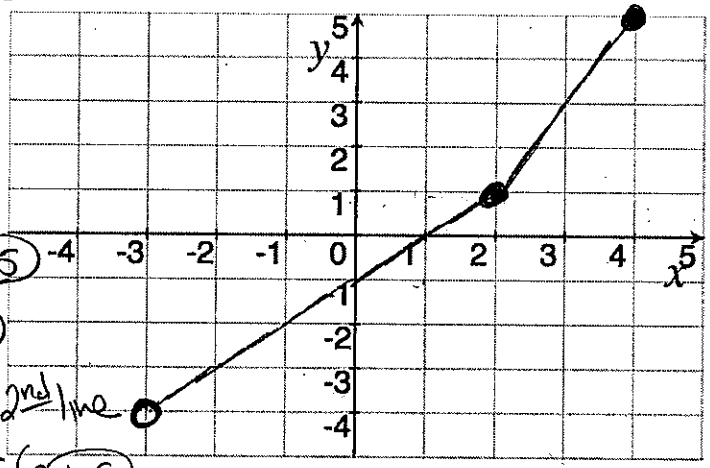
(e) [1] (§1.1 #48) Identify the y intercept. of f
-1 +1

(f) [3] (§2.1 #32) Find the equation for f.
vertex $(2, -3)$ so $y = a(x-h)^2 + k$ +.5
$\Rightarrow y = a(x-2)^2 - 3$ +1
since passes thru $(0,-1)$ +.5 we know $-1 = a(0-2)^2 - 3$
$\Rightarrow -1 = 4a - 3 \Rightarrow 2 = 4a$
$\Rightarrow a = \frac{1}{2}$ +.5

So $y = \frac{1}{2}(x-2)^2 - 3$ +.5

15

4. Let $h(x) = \begin{cases} x-1 & -3 < x \le 2 \\ 2x-3 & 2 < x \le 4 \end{cases}$



(a) [2] (WebHW3 #17 & WebHW6 #17)
Estimate the following *if* possible:

i. $h(0)$ $-3 < 0 \le 2$ (+.5) use 1st line
$0-1 = -1$ (+.5)

ii. $(h+h)(3)$
$= h(3)+h(3)$ (+.5) use 2nd line
$= (2(3)-3)+(2(3)-3) = 6$ (+.5)

(b) [3] (WebHW3 #18)
Graph h.
domain (+.5)

slope or x-intercept (+.5) get it (+.5)

slope or x-intercept (+.5) get it (+.5)

(c) [2] (Quiz2 #3)
What is the range of h?

$(-4, 5]$ (+.5) (+.5)

y values (+.5) bands (+.5) or $-4 < y \le 5$

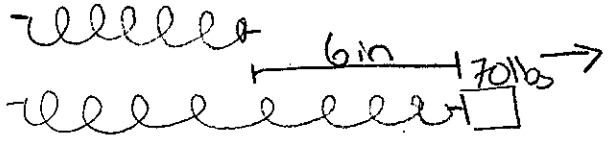
5. [2] (WebHW7 #16) Divide $\frac{3i}{9-6i}$

$\frac{3i}{9-6i}\cdot\frac{9+6i}{9+6i} = \frac{27i+18i^2}{81+54i-54i-36i^2} = \frac{-18+27i}{81+36} = \frac{-18}{117}+\frac{27}{117}$

$\approx -.15+.23$

(+.5) FOIL (+.5) $i^2 = -1$ (+.5) get it (+.5)

6. [3] (Quiz1 #4) The distance that a spring will stretch varies directly as the force applied to the spring. A force of 70 pounds is needed to stretch a spring 6 inches. What force is required to stretch the spring 20 inches?



d = distance spring stretches
F = force applied

$d = kF$ (+1)

(+1) $6 = k\,70 \Rightarrow k = \frac{6}{70} = \frac{3}{35}$

So find F so that d = 20 inches

(+.5) $20 = \frac{3}{35}\cdot F$

(+.5) $F = 20\cdot\frac{35}{3} \approx 233\text{lb}$

12

7. Let $p(x) = (x+3)^2 + 1$

(a) [1] (§1.3 #32) Find $p(2+h)$. $= (2+h+3)^2 + 1$

$= [(2+h)+3]^2 + 1$

(b) [2] (§2.1 #92b) Find the real or complex roots of $p(x)$.

roots when $y=0$ (+.5)

$0 = (x+3)^2 + 1$

$-1 \quad -1$

$-1 = (x+3)^2$ alg (+1)

$\pm\sqrt{-1} = x+3$ (+.5)

$-3 \quad -3$

$-3 \pm \sqrt{-1} = x$

$-3 \pm i = x$

or

roots when $y=0$ (1.5)

$0 = (x+3)^2 + 1$ alg (+1)

$0 = x^2 + 6x + 9 + 1$ quad formula (+1)

$0 = x^2 + 6x + 10$

$x = \frac{-6 \pm \sqrt{36 - 4(1)(10)}}{2(1)}$

8. [5] You have a 375g block of iron ore and want to know how much iron is in it. You take a 100 gram sample of iron ore and found it contained 34g of iron. About how many grams of iron are in the original 375g block? Write a function whose output gives you the percentage (in decimal form) of iron as a function of how how many grams of pure iron are added to it (which can be done when you melt the iron ore down).

start (+.5)

$\frac{34\text{g iron}}{100\text{g block}} \cdot 375\text{g block} = 127.5\text{g of pure iron}$

ratio/dim analysis (+.5) (+.5)

(+.5) % of pure iron in block $= \frac{\text{amount of pure iron}}{\text{total grams}}$

(+1) $= \frac{\text{original pure iron} + \text{added pure iron}}{\text{total grams}}$

$= \frac{127.5 + x}{375 + x}$ (+.5) (+.5)

(+1) where x = the grams of pure iron added

25
20
45

8