

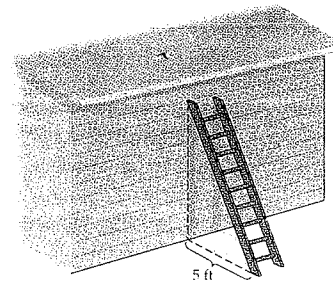
cal language, and finding an equation that will produce the solution to the problem. You need not solve this equation.

5. A worker gets an 8% pay raise and now makes \$1600 per month. What was the worker's old salary?
6. A merchant has 5 pounds of mixed nuts that cost \$30. He wants to add peanuts that cost \$1.50 per pound and cashews that cost \$4.50 per pound to obtain 50 pounds of a mixture that costs \$2.90 per pound. How many pounds of peanuts are needed?
7. The diameter of a circle is 16 cm. By what amount must the radius be decreased to decrease the area by  $48\pi$  square centimeters?
8. A corner lot has dimensions 25 by 40 yards. The city plans to take a strip of uniform width along the two sides bordering the streets to widen these roads. How wide should the strip be if the remainder of the lot is to have an area of 844 square yards?

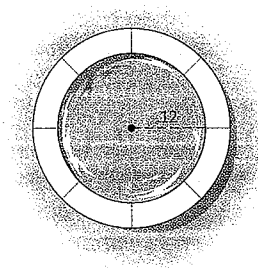
In the remaining exercises, solve the applied problems.

9. You have already invested \$550 in a stock with an annual return of 11%. How much of an additional \$1100 should be invested at 12% and how much at 6% so that the total return on the entire \$1650 is 9%?
10. If you borrow \$500 from a credit union at 12% annual interest and \$250 from a bank at 18% annual interest, what is the *effective annual interest rate* (that is, what single rate of interest on \$750 would result in the same total amount of interest)?
11. A radiator contains 8 quarts of fluid, 40% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 60% antifreeze?
12. A radiator contains 10 quarts of fluid, 30% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 40% antifreeze?
13. An airplane flew with the wind for 2.5 hours and returned the same distance against the wind in 3.5 hours. If the cruising speed of the plane was a constant 360 mph in air, how fast was the wind blowing? [Hint: If the wind speed is  $r$  miles per hour, then the plane travels at  $(360 + r)$  mph with the wind and at  $(360 - r)$  mph against the wind.]
14. A train leaves New York for Boston, 200 miles away, at 3:00 P.M. and averages 75 mph. Another train leaves Boston for New York on an adjacent set of tracks at 5:00 P.M. and averages 45 mph. At what time will the trains meet?
15. The average of two real numbers is 41.125, and their product is 1683. What are the numbers? [Hint: See Example 1.]

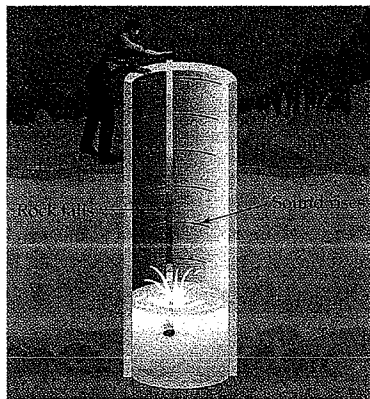
16. A rectangle is twice as long as it is wide. If it has an area of 24.5 inches, what are its dimensions? [Hint: See Example 2.]
17. Two cars leave a gas station at the same time, one traveling north and the other south. The northbound car travels at 50 mph. After 3 hours, the cars are 345 miles apart. How fast is the southbound car traveling?
18. A student leaves the university at noon, bicycling south at a constant rate. At 12:30 P.M., a second student leaves the same point and heads west, bicycling 7 mph faster than the first student. At 2:00 P.M., they are 30 miles apart. How fast is each one going?
19. A 13-foot-long ladder leans on a wall, as shown in the figure. The bottom of the ladder is 5 feet from the wall. If the bottom is pulled out 3 feet farther from the wall, how far does the top of the ladder move down the wall? [Hint: Draw pictures of the right triangle formed by the ladder, the ground, and the wall before and after the ladder is moved. In each case, use the Pythagorean Theorem to find the distance from the top of the ladder to the ground.]



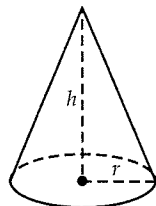
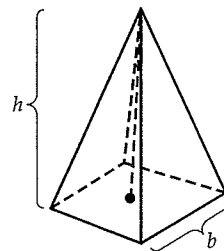
20. A 15-foot-long pole leans against a wall. The bottom is 9 feet from the wall. How much farther should the bottom be pulled away from the wall so that the top moves the same amount down the wall?
21. A concrete walk of uniform width is to be built around a circular pool, as shown in the figure. The radius of the pool is 12 meters, and enough concrete is available to cover  $52\pi$  square meters. If all the concrete is to be used, how wide should the walk be?



22. In Example 6 of Section 1.4, how many can openers must be produced to have an average cost per can opener of \$3?
23. A rock is dropped into a well, and 3 seconds later the sound of its splash is heard. How deep is the well? Assume that sound travels at 1100 feet per second and that an object falls a distance of  $16t^2$  feet in  $t$  seconds. [Hint: Let  $t$  be the time it takes for the rock to fall to the water. Then it takes  $3 - t$  seconds for the sound of its splash to rise to the top (why?). Both the rock and the sound travel the same distance.]

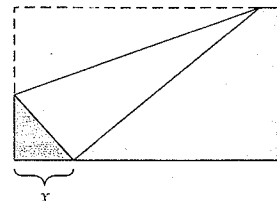


24. A group of homeowners are to share equally in the \$210 cost of repairing a bus-stop shelter near their homes. At the last moment, two members of the group decide not to participate, and this raises the share of each remaining person by \$28. How many people were in the group at the beginning?
25. Red Riding Hood drives the 432 miles to Grandmother's house in 1 hour less than it takes the Wolf to drive the same route. Her average speed is 6 mph faster than the Wolf's average speed. How fast does each drive?
26. To get to work, Sam jogs 3 kilometers to the train and then rides the remaining 5 kilometers. If the train goes 40 kilometers per hour faster than Sam's constant rate of jogging and the entire trip takes 30 minutes, how fast does Sam jog?
27. The dimensions of a rectangular box are consecutive integers. If the box has volume of 13,800 cubic centimeters, what are its dimensions?
28. Find a real number that exceeds its cube by 2.
29. The surface area  $S$  of the right circular cone in the figure below is given by  $S = \pi r \sqrt{r^2 + h^2}$ . What radius should be used to produce a cone of height 5 inches and surface area 100 square inches?
30. The surface area of the right square pyramid in the figure below is given by  $S = b \sqrt{b^2 + 4h^2}$ . If the pyramid has height 10 feet and surface area 100 square feet, what is the length of a side  $b$  of its base?
31. Suppose that the open-top box being made from a sheet of cardboard in Example 9 is required to have at least one of its dimensions *greater* than 18 inches. What size square should be cut from each corner?
32. A homemade loaf of bread turns out to be a perfect cube. Five slices of bread, each .6 inch thick, are cut from one end of the loaf. The remainder of the loaf now has a volume of 235 cubic inches. What were the dimensions of the original loaf?
33. A rectangular bin with an open top and volume of 38.72 cubic feet is to be built. The length of its base must be twice the width, and the bin must be at least 3 feet high. Material for the base of the bin costs \$12 per square foot, and material for the sides costs \$8 per square foot. If it costs \$538.56 to build the bin, what are its dimensions?



### Thinker

34. One corner of an  $8.5 \times 11$  inch piece of paper is folded over to the opposite side, as shown in the figure. The area of the darkly shaded triangle at the lower left is 6 square inches and we want to find the length  $x$ .
- (a) Take a piece of paper this size and experiment. Approximately, what is the largest value  $x$  could have (and still have the paper look like the figure)? With this value of  $x$ , what is the approximate area of the triangle? Try some other possibilities.
- (b) Now find an exact answer by constructing and solving a suitable equation. Explain why one of the solutions to the equation is not an answer to this problem.



11.  $y = -(x - 1)^2 + 2$       12.  $h(x) = -x^2 + 1$   
 13.  $f(x) = x^2 - 6x + 3$       14.  $g(x) = x^2 + 8x - 1$   
 15.  $h(x) = x^2 + 3x + 6$       16.  $f(x) = x^2 - 5x - 7$   
 17.  $y = 2x^2 + 12x - 3$       18.  $y = 3x^2 + 6x + 1$   
 19.  $f(x) = -x^2 + 8x - 2$       20.  $g(x) = -x^2 - 6x + 4$

In Exercises 21–24, find

- (a) The difference quotient of the functions;  
 (b) The vertex of the function's graph;  
 (c) The value of the difference quotient at the  $x$ -coordinate of the vertex.

21.  $f(x) = -x^2 + x$       22.  $g(x) = 2x^2 - x - 1$   
 23.  $f(x) = -2x^2 + 2x - 1$       24.  $g(x) = -3x^2 + 4x + 5$

25. The graph of the quadratic function  $g$  is obtained from the graph of  $f(x) = x^2$  by vertically stretching it by a factor of 2 and then shifting vertically 5 units downward. What is the rule of the function  $g$ ? What is the vertex of its graph?

26. The graph of the quadratic function  $g$  is obtained from the graph of  $f(x) = x^2$  by shifting it horizontally 4 units to the left, then vertically stretching it by a factor of 3, and then shifting vertically 2 units upward. What is the rule of the function  $g$ ? What is the vertex of its graph?

27. If the graph of the quadratic function  $h$  is shifted vertically 4 units downward, then shrunk by a factor of  $1/2$ , and then shifted horizontally 3 units to the left, the resulting graph is the parabola  $f(x) = x^2$ . What is the rule of the function  $h$ ? What is the vertex of its graph?

28. If the graph of the quadratic function  $h$  is shifted vertically 3 units upward, then reflected in the  $x$ -axis, and then shifted horizontally 5 units to the right, the resulting graph is the parabola  $f(x) = x^2$ . What is the rule of the function  $h$ ? What is the vertex of its graph?

In Exercises 29–32, find the rule of the quadratic function whose graph satisfies the given conditions.

29. Vertex at  $(0, 0)$ ; passes through  $(2, 12)$   
 30. Vertex at  $(0, 1)$ ; passes through  $(2, -7)$   
 31. Vertex at  $(2, 5)$ ; passes through  $(-3, 80)$   
 32. Vertex at  $(4, 1)$ ; passes through  $(2, -11)$   
 33. Find the number  $b$  such that the vertex of the parabola  $y = x^2 + bx + c$  lies on the  $y$ -axis.  
 34. Find the number  $c$  such that the vertex of the parabola  $y = x^2 + 8x + c$  lies on the  $x$ -axis.  
 35. If the vertex of the parabola  $f(x) = x^2 + bx + c$  is at  $(2, 4)$ , find  $b$  and  $c$ .  
 36. If the vertex of the parabola  $f(x) = -x^2 + bx + 8$  has second coordinate 17 and is in the second quadrant, find  $b$ .

37. The Leslie Lahr Luggage Company has determined that its profit on its Luxury ensemble is given by

$$p(x) = 1600x - 4x^2 - 50,000,$$

where  $x$  is the number of units sold.

- (a) What is the profit on 50 units? On 250 units?  
 (b) How many units should be sold to maximize profit? In that case, what will be the profit on each unit?

38. On the basis of data from past years, a consultant informs Bob's Bicycles that their profit from selling  $x$  bicycles is given by the function

$$p(x) = 250x - x^2/4 - 15,000.$$

- (a) How much profit do they make by selling 100 bicycles? By selling 400 bicycles?  
 (b) How many bicycles should be sold to maximize profit? In that case, what will be the profit per bicycle?

39. During the Civil War, the standard heavy gun for coastal artillery was the 15-inch Rodman cannon, which fired a 330-pound shell. If one of these guns is fired from the top of a 50-foot-high shoreline embankment, then the height of the shell above the water (in feet) can be approximated by the function

$$p(x) = -.0000167x^2 + .23x + 50,$$

where  $x$  is the horizontal distance (in feet) from the foot of the embankment to a point directly under the shell. How high does the shell go, and how far away does it hit the water?

40. The Golden Gate Bridge is supported by two huge cables strung between the towers at each end of the bridge. The function

$$f(x) = .0001193x^2 - .50106x + 526.113$$

gives the approximate height of the cables above the roadway at a point on the road  $x$  feet from one of the towers. The cables touch the road halfway between the two towers. How far apart are the towers?

41. The braking distance (in meters) for a car with excellent brakes on a good road with an alert driver can be modeled by the quadratic function  $B(s) = .01s^2 + .7s$ , where  $s$  is the car's speed in kilometers per hour.  
 (a) What is the braking distance for a car traveling 30 kilometers per hour? For one traveling 100 kilometers per hour?  
 (b) If the car takes 60 meters to come to a complete stop, what was its speed?

42. Jack throws a baseball. Its height above the ground (in feet) is given by

$$h(x) = -.0013x^2 + .26x + 5,$$

where  $x$  is the distance (in feet) from Jack to a point on the ground directly below the ball.

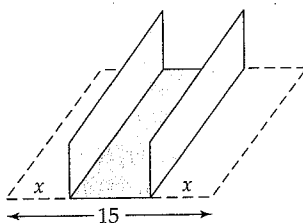
- (a) How far from Jack is the ball when it reaches the highest point on its flight? How high is the ball at that point?  
 (b) How far from Jack does the ball hit the ground?

In Exercises 43–46, use the formula for the height  $h$  of an object that is traveling vertically (subject only to gravity) at time  $t$ :

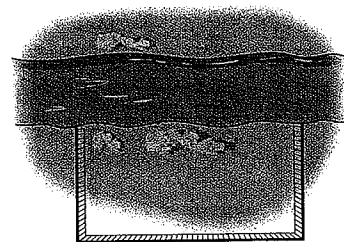
$$h = -16t^2 + v_0t + h_0,$$

where  $h_0$  is the initial height and  $v_0$  is the initial velocity;  $t$  is measured in seconds and  $h$  in feet.

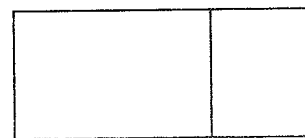
43. A ball is thrown upward from the top of a 96-foot-high tower with an initial velocity of 80 feet per second. When does the ball reach its maximum height and how high is it at that time?  
 44. A rocket is fired upward from ground level with an initial velocity of 1600 feet per second. When does it attain its maximum height and what is that height?  
 45. A ball is thrown upward from a height of 6 feet with an initial velocity of 32 feet per second. Find its maximum height.  
 46. A bullet is fired upward from ground level with an initial velocity of 1500 feet per second. How high does it go?  
 47. The sum of the height  $h$  and the base  $b$  of a triangle is 30. What height and base will produce a triangle of maximum area?  
 48. A gutter is to be made by bending up the edge of a 15-inch-wide piece of aluminum. What depth should the gutter be to have the maximum possible cross-sectional area?



49. A field bounded on one side by a river is to be fenced on three sides so as to form a rectangular enclosure. 200 feet of fencing is to be used, what dimensions will yield an enclosure of the largest possible area?



50. A rectangular box (with top) has a square base. The sum of the lengths of its 12 edges is 8 feet. What dimensions should the box have so that its surface area is as large as possible?  
 51. A gardener wants to use 130 feet of fencing to enclose a rectangular garden and divide it into two plots, as shown in the figure. What is the largest possible area for such a garden?



52. A rectangular garden next to a building is to be fenced on three sides. Fencing for the side parallel to the building costs \$80 per foot, and material for the other two sides costs \$20 per foot. If \$1800 is to be spent on fencing, what are the dimensions of the garden with the largest possible area?  
 53. At Middleton Place, a plantation near Charleston, South Carolina, there is a "joggling board" that was once used for courting. A young girl would sit at one end, her suitor at the other end, and her mother in the center. The mother would bounce on the board, thus causing the girl and suitor to move closer together. A joggling board is 8 feet long and an average mother sitting at its center causes