MIDTERM 2 MATH 111 Practice

NAME: Key

1. [10] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

Let f and g be functions.

T F $f(x+y) = f(x) + f(y)$

T F $(f-g)(x) = f(x) - g(x)$

T F $f(g(x)) = g(f(x))$

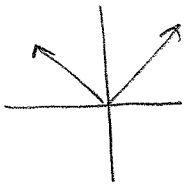
T F $(fg)(x) = (gf)(x)$ $(fg)(x) = f(x)g(x) = g(x)f(x) = (gf)(x)$

T F $f(\frac{x}{y}) = \frac{f(x)}{f(y)}$

T F $\log(\log(10)) = 0.$ $\log(1) = 0$

T F $\frac{\log v}{\log w} = \log v - \log w$ for $v, w > 0$ $\log \frac{v}{w} = \log v - \log w$

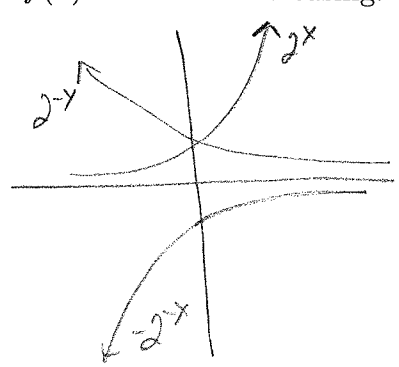
T F A function is 1-1 if and only if any vertical line passes through the graph of f at most once. → vert. line test is for functions - not 1-1 functions.



T F If $g(x) = 6|x - 5.24| + \log_{2.7} 8$, then $g(x)$ has an inverse function.

vert stretch by 6
moved up by $\log_{2.7} 8$
horiz shifted right
does not pass the horiz. line test

T F The function f with the rule $f(x) = -2^{-x}$ is increasing.



FILL-IN-THE-BLANK: Write the most appropriate answer in the space given. Correct answers will *not* get credit without supporting work.

2. [2] Given $f(2) = 6$ and $g(x) = 3x - 7$, $f(g(3))$ is 6.

$$f(g(3)) = f(3(3)-7) = f(9-7) = f(2) = 6$$

3. [3] Let f be defined by $f(x) = \dfrac{6x-1}{1-3x}$. Assuming the inverse of f exists, it is: _____.

$$x = \frac{6g(x)-1}{1-3g(x)} \Rightarrow x - 3xg(x) = 6g(x) - 1$$

$$x + 1 = 6g(x) + 3xg(x)$$

$$g(x) = \frac{x+1}{6+3x}$$

4. [2] If $f(2) = 6$ and the graph of g looks like the graph of f stretched vertically by a factor of 2 and shifted down 1, then $g(2)$ is 11.

$$g(x) = 2f(x) - 1$$

vert stretch vert shift

so $g(2) = 2f(2) - 1$

$= 2 \cdot 6 - 1 = 11$

5. [1] If f and g are inverses and the range of f is $(-\infty, 67]$, then the domain of g is: $(-\infty, 67]$

range of f = domain of g

LONG ANSWERS: Show all your work and circle you final answer. Correct answers will *not* get credit without supporting work.

6. [1] Given $-x = \frac{2xy}{2y-1}$, solve for y.

$-2xy + x = 2xy$

$x = 4xy$

$\frac{1}{4} = \frac{\cancel{x}}{4\cancel{x}} = y$

7. [2] Define the rule of the function log. Let $v > 0$, then $\log v = u$ when $10^u = v$.

8. [4] Assume b, x,$y > 0$, simplify the following:

$\frac{(b^x)^{x-1}}{b^{-x}}$

$\frac{b^{x(x-1)}}{b^{-x}} = \frac{b^{x^2-x}}{b^{-x}} = b^{x^2-x} b^{x}$

$= b^{x^2-x+x} = b^{x^2}$

$\frac{\sqrt[3]{x^2}(y^2)^{\frac{3}{2}}}{x^{\frac{2}{3}}y^2} = \frac{\cancel{x^{\frac{2}{3}}} y^3}{\cancel{x^{\frac{2}{3}}} y^2}$

$= y^3 y^{-2} = y$

9. Write the given expression as a single logarithm.

$2\ln 2x - 3(\ln x^2 + \ln x)$

$\ln (2x)^2 - 3(\ln x^2 \cdot x)$

$\ln 4x^2 - 3\ln x^3$

$\ln 4x^2 - \ln (x^3)^3$

$\ln 4x^2 - \ln x^9$

$\ln \frac{4x^2}{x^9} = \ln \frac{4}{x^7}$

$(x^3)^3 = x^3 \cdot x^3 \cdot x^3$

$3 - \log_6(36y)$

$\log_6 6^3 - \log_6 36y$

$\log_6 \frac{6^3}{6^2 y}$

$\log_6 \frac{6}{y}$

10. [3] Find x in the following:

$2^{4x-1} = 3^{1-x}$)

$(4x-1)\ln 2 = (1-x)\ln 3$

$4x\ln 2 - \ln 2 = \ln 3 - x\ln 3$

$4x\ln 2 + x\ln 3 = \ln 3 + \ln 2$

$x = \dfrac{\ln 3 + \ln 2}{4\ln 2 + \ln 3}$

$5^x = 2$

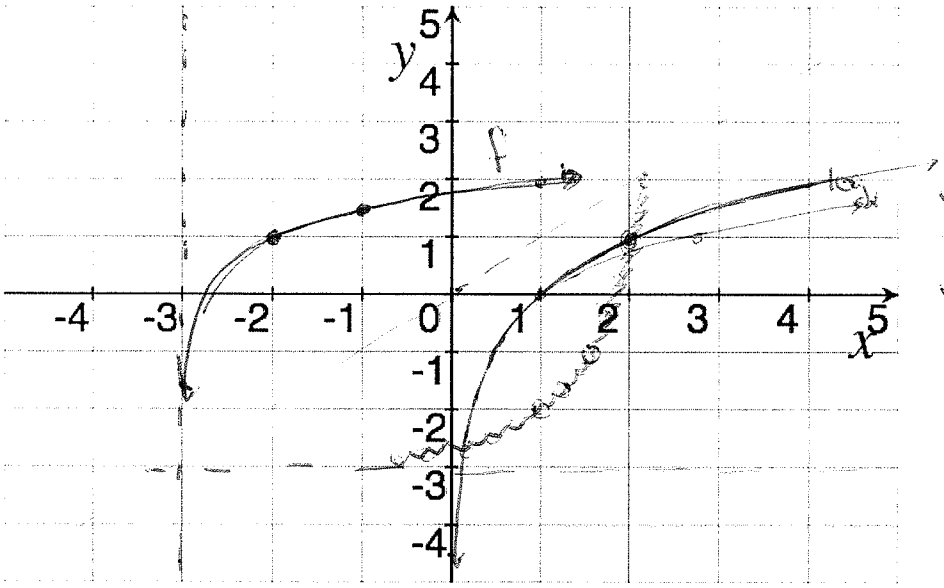
$\log_5(\cdot)$ both

$\log_5 5^x = \log_5 2$

$x = \log_5 2$

note: there are many answers that look diff but are equivalent

11. [4] List the transformations needed to transform the graph of $h(x) = \log_2 x$ to the graph of $f(x) = \frac{1}{2}\log_2(x+3) + 1$. Then graph f and its *inverse*.



vert stretch by $\frac{1}{2}$
vert shift up by 1
horiz shift left by 3

the points $\left\{\begin{matrix}(-2,1)\\(-1,\frac{3}{2})\\(1,2)\end{matrix}\right\}$ are on f so $\left\{\begin{matrix}(1,-2)\\(\frac{3}{2},-1)\\(2,1)\end{matrix}\right\}$ are on the inverse

the ~~two~~ give the graph of the inverse

13. [] Anne has \$20,000 in student loans at the end of her college education with 8% interest compounded quarterly. Anne chooses not to make payments after graduation since she has trouble

(a) How much money will Anne owe after 2 years?

$$20{,}000\left(1+\frac{.08}{4}\right)^{4\cdot 2} = 20{,}000\left(1+\frac{8}{400}\right)^{8}$$
$$= 20{,}000\left(1+\frac{2}{100}\right)^{8}$$

(b) How long will it take for Anne's loan to double?

find t so that

$$40{,}000 = 2\,10{,}000\left(1+\frac{.08}{4}\right)^{4t}$$

$$\Rightarrow \quad 2 = \left(1+\frac{2}{100}\right)^{4t}$$

$$\ln 2 = 4t \ln\left(\frac{51}{50}\right)$$

$$t = \frac{\ln 2}{4\ln\left(\frac{51}{50}\right)}$$

note: other possible word problems could make use of anything from §5.2

like

$$Pa^{x} = y$$
$$P\left(1+\frac{r}{2}\right)^{2t} = y$$
$$Pe^{rt} = y$$
$$c\left(\frac{1}{2}\right)^{\frac{x}{h}} = y.$$

12. Let $f(x) = 3x + 1$ and $g(x) = \sqrt{x-1}$. Find the following, and specify the domain of each one.

- [2] $(f-g)(x)$ $= f(x) - g(x) = 3x+1 - \sqrt{x-1}$

  Domain: x so that $x-1 \geq 0$

  $\Rightarrow x \geq 1$ or $[1, \infty)$

- [2] $(fg)(x)$ $= f(x)g(x) = (3x+1)\sqrt{x-1}$

  Domain: x so that $x-1 \geq 0$

  $\Rightarrow x \geq 1$ or $[1, \infty)$

- [3] $\frac{g}{f}(x)$ $= \frac{g(x)}{f(x)} = \frac{\sqrt{x-1}}{3x+1}$

  Domain: x so that $x-1 \geq 0$ and $3x+1 \neq 0$

  $\Rightarrow x \geq 1$ $\quad x \neq -\frac{1}{3}$ already taken into account

  or $[1, \infty)$

- [1] $g(f(x))$

  $= g(3x+1) = \sqrt{(3x+1)-1}$

  Domain: x so that $3x+1-1 \geq 0$ or $[0, \infty)$

  $3x \geq 0$

  $x \geq 0$

- [3] $f(g(x))$

  $= f(\sqrt{x-1}) = 3\sqrt{x-1} + 1$

  Domain: x so that $x-1 \geq 0$ or $[1, \infty)$

  $x \geq 1$