Math 111
Graph Transformations Worksheet

Review:

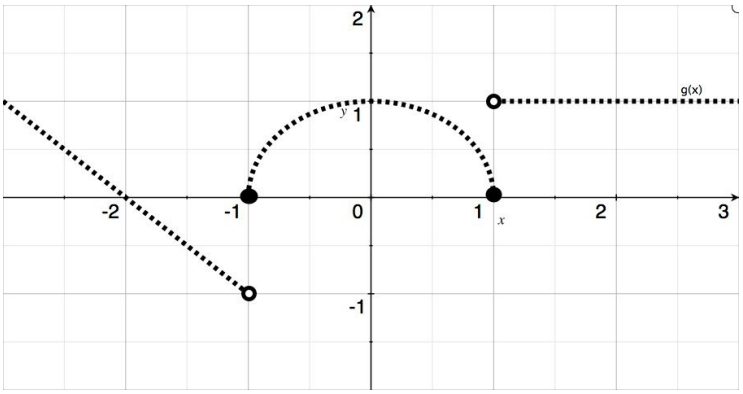
1. Finish the following statements.

   (a) Let the graph of the function f be given and $g(x) = af(x)$, where a is a positive constant. Then the graph of g is exactly the same as the graph of f, but

   (b) Let the graph of the function f be given and $g(x) = f(x)+b$, where b is a negative constant. Then the graph of g is exactly the same as the graph of f, but

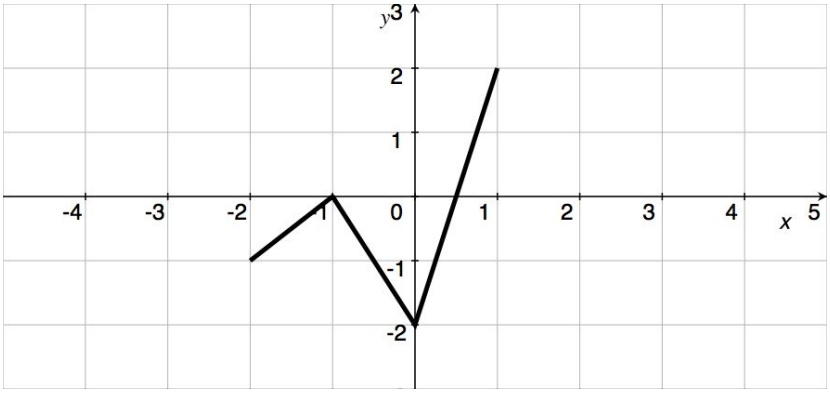
   (c) Let the graph of the function f be given and $g(x) = af(x)+b$, where a and b are constants. Then the graph of g is obtained from the graph of f, by first

2. The graph of the function g is below. Draw the graph of $m(x) = \frac{1}{2}g(x) + 1$ and $n(x) = -g(x) + \frac{1}{2}$.

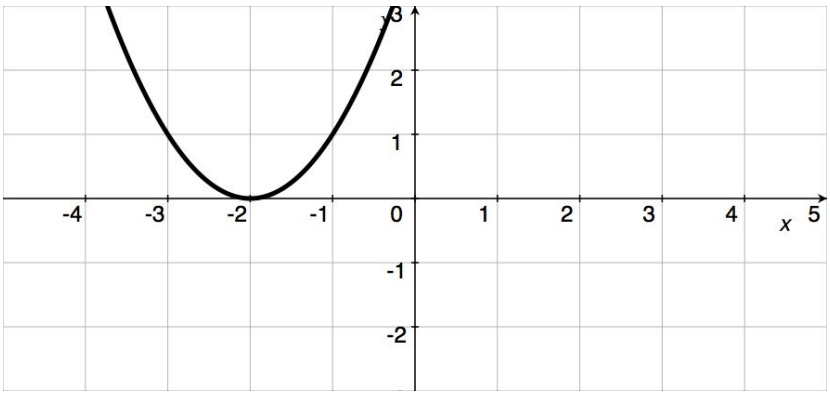


3. Finish the following statement: Let the graph of the function f be given and $g(x) = f(x + c)$, where c is a positive constant. Then the graph of g is exactly the same as the graph of f, but

4. The graph of the function f is below. Draw the graph of $g(x) = f(x + 2)$ and $h(x) = f(x - 3)$.



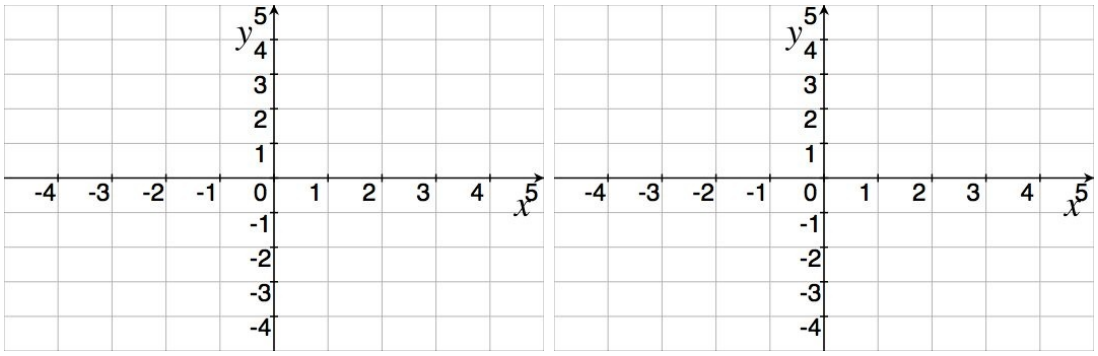
5. The graph of the function g is below. Can you write a rule for g. (Hint: g is the horizontal shift of the function $f(x) = x^2$.)



Flips

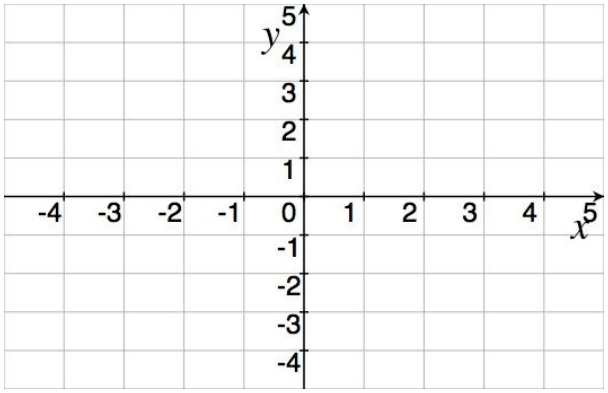
1. Using a table of values or a graphing calculator sketch the following function with f and g on one axis, and h and j on another.

$$f(x) = \sqrt{x}, g(x) = \sqrt{-x}, h(x) = 2x + 3, j(x) = 2(-x) + 3$$



2. How would you describe the relationship between the graphs of f and g, or the relationship between the graphs of h and j? In general, what is the relationship between the graphs of $m(x)$, and $m(-x)$? (This behavior is called a **reflection over the y-axis**.)

3. Using a table of values or your graphing calculator, graph $f(x) = x^2$, and $g(x) = (-x + 2)^2$ on the same axis.



4. Refer to # 3 for $f(x)$ and $g(x)$. According to all the work done previously, the graph of $g(x)$ is the same as the graph of $f(x)$ with graph transformations. Which of the following two possible order of graph transformations match with the graph of $g(x)$ above? Hint: Just like we did in class, graph each set of transformations below and see what matches with your graph above.

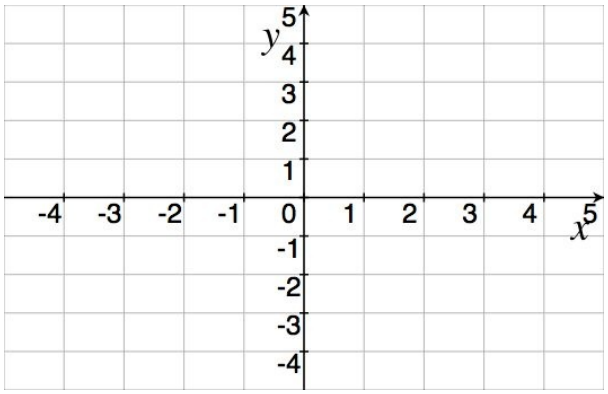
   - a horizontal shift to the left 2 followed by a flip over the y-axis
   - a flip over the y-axis followed by a horizontal shift to the left 2

5. As in our work with vertical transformations, it matters if we shift first and then flip, or flip and then stretch. Note that when we are working inside the function (and thus doing horizontal transformations) it is *backwards* from the vertical we are aquatinted with, now, we shift first!.

Vertical and Horizontal Combinations

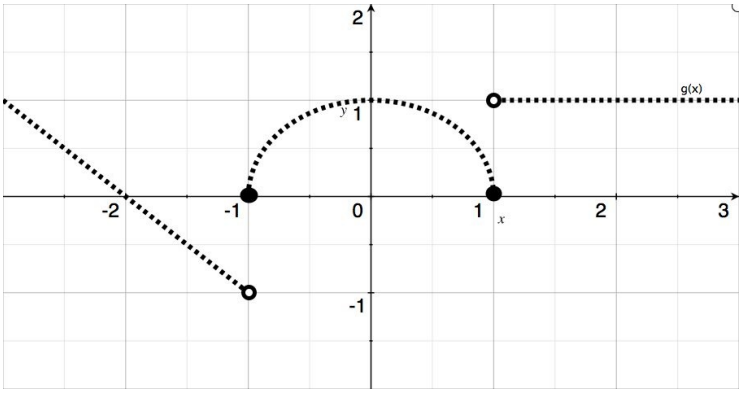
1. Lets combine vertical graph transformations with our new horizontal tricks. Use a table of values or a graphing calculator to sketch the following on the same set of axes:

$$f(x) = x^2, g(x) = (x+1)^2 - 1, h(x) = (x-2)^2 - 2$$



2. Thinking in terms of graph transformations, given $f(x) = x^2$, to get $g(x) = (x-2)^2 - 1$, would you shift vertically first or second? Does it matter?

3. Given the graph $g(x)$ below, draw $f(x) = g(x+2) - 1$



4. Using a table of values or a graphing calculator, graph the following functions on the same axis:

$$f(x) = \sqrt{x}, g(x) = 2\sqrt{x+1}, h(x) = 2\sqrt{x+1} - 3, j(x) = 2\sqrt{-x+1} - 3$$

5. From above we know that horizontal shifts must be done before horizontal flips. Recall from yesterday that vertical stretches and flips should be done before vertical shifts. Consider the graph transformations that would take $f(x) = \sqrt{x}$, to $j(x) = 2\sqrt{x+1} - 3$. Does it matter if you do all the vertical transformations first or second? Hint: write down the necessary graph transformations and perform first all vertical transformations and then horizontal transformations. Perform the transformations again on $f(x)$, doing first the horizontal and then the vertical and compare the results.

6. The above is only an example, but it is true in general, the vertical transformations can be performed before or after the horizontal transformations and you will end up with the same graph. The important part to remember is that when doing the vertical transformations, you have to stretch vertically, before any shifts up or down. Similarly, when performing the horizontal transformations the horizontal shifts must come before any flips over the y-axis.