Practice Final MATH 111 Partial Solutions

NAME:

[11] Let f & g, be functions, and x & y be real numbers.

T $(fg)(x) = (gf)(x)$
Note $f(x)g(x) = g(x)f(x)$.

F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$
Note sometimes $\frac{f(x)}{g(x)} \neq \frac{g(x)}{f(x)}$.

F $x^2 = y$ defines x as a function of y
Note there are two values of x *which satisfy* $x^2 = y$ *when* $y > 0$.

T $x - 2$ is a factor of $\frac{1}{2}x^4 - 2x^2 + x - 2$
Use the factor theorem and the fact that $\frac{1}{2}(2^4) - 2(2^2) + 2 - 2 = 0$.

F $\log(\log(e)) = 0$.
Note $\log(\log(e)) \approx -0.36221$.

T The diameter of a circle varies directly with the radius.
Note $d = 2r$.

Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. [2] Define $\log x = y$

   $\log x = y$ *exactly when* $10^y = x$. *(Alternatively,* $\log x$ *is the exponent you need to raise* 10 *to in order to get* y*.)*

2. [2] Which of the following may be a graph of a polynomial of degree five with a positive leading coefficient?

3. [2] Which of the following is a graph of an even function?

4. [3] If $f(x)$ is an even function, $f(2) = 6$, and $g(x) = \frac{1}{2}f(2x) - \frac{1}{3}$, what is $g(-1)$?

   $g(-1) = \frac{1}{2}f(-2) - \frac{1}{3} = \frac{1}{2} \cdot 6 - \frac{1}{3} = 3 - \frac{1}{3} = \frac{8}{3}$, since $f(-2) = 6$.

5. [3] Find the equation for a line that is perpendicular to the line with end points $(51, 60)$ and $(53, 50)$.

   The slope of the original line is $m_1 = \frac{60-50}{51-53} = \frac{10}{-2} = -5$. Thus, our new line will have slope $m_2 = \frac{1}{5}$. Such a line is given by $y = \frac{1}{5}x$.

6. [3] Given $kx^2 + 5x - 2 = 0$, what does k have to be to ensure 2 real solutions? Give answer in interval notation.

   Notice if $k = 0$, then we'll have a linear equation, which would only have one solution, thus $k \neq 0$.

   We note that this polynomial has two real solutions when the descriminant $5^2 - 4 \cdot k \cdot -2 > 0$. Thus, $8k > -25$, so $k > -25/8$. In interval notation, the solution is expressed by $\left(-\frac{25}{8}, 0\right) \cup (0, \infty)$.

7. [3] Let g be the function defined by the rule $g(x) = |x^2 + 3x - 6|$. Find x such that $g(x) = 2x$.

if $\lvert x^2+3x-6\rvert \geq 0$		if $\lvert x^2+3x-6\rvert < 0$
$\lvert x^2+3x-6\rvert = x^2+3x-6$		$\lvert x^2+3x-6\rvert = -(x^2+3x-6)$

So

$g(x) = 2x$		$g(x) = 2x$
$x^2+3x-6 = 2x$		$-(x^2+3x-6) = 2x$
$x^2+x-6 = 0$	or	$x^2+3x-6 = -2x$
$(x+3)(x-2) = 0$		$x^2+5x-6 = 0$
$x = -3$ or $x = 2$		$(x+6)(x-1) = 0$
		$x = -6$ or $x = 1$

Checking:

if $x = -3$	if $x = 2$	if $x = -6$	if $x = 1$
$g(-3)?2(-3)$	$g(2)?2(2)$	$g(-6)?2(-6)$	$g(1)?2(1)$
$\lvert(-3)^2+3(-3)-6\rvert?-6$	$\lvert(2)^2+3(2)-6\rvert?4$	$\lvert(-6)^2+3(-6)-6\rvert?-12$	$\lvert(1)^2+3(1)-6\rvert?2$
$\lvert-6\rvert?-6$	$\lvert 4\rvert?4$	$\lvert 12\rvert?-12$	$\lvert-2\rvert?2$
$6 \neq -6$	$4 = 4$	$12 \neq -12$	$2 = 2$

8. Given $f(x) = \frac{1}{1-x} - 2$:

- [5] Compute the difference quotient. Recall the difference quotient is:

$$f(x) = \frac{f(x+h) - f(x)}{h}$$

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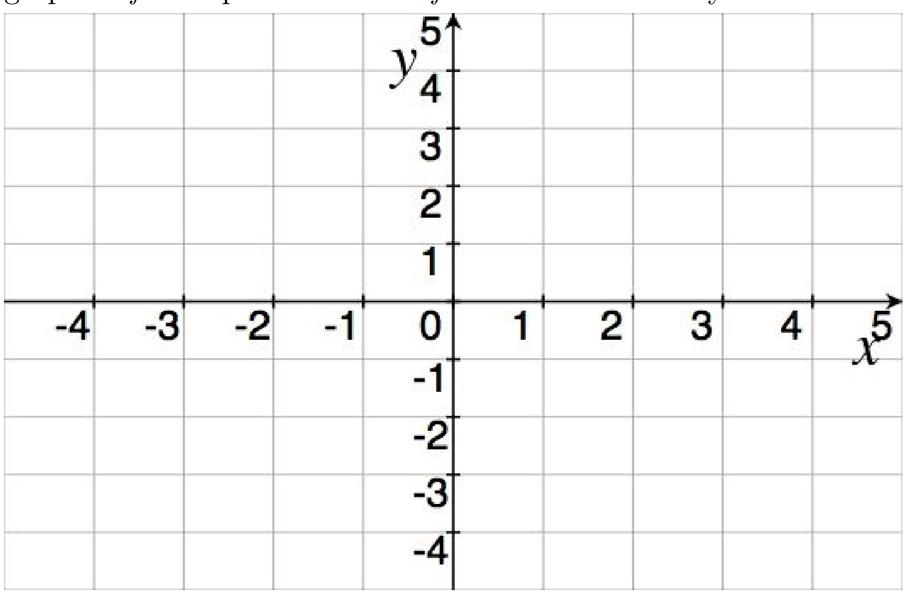
- [2] List the transformations needed to transfrm the graph of $h(x) = \frac{1}{x}$ into the graph of f. Graph both h and f. Be sure to identify which one is which.



- [2] Algebraically find the inverse of $f(x)$.

9. [15] Let $f(x) = \sqrt{7x-3}$, and $g(x) = 3x - 7$. Find the following, and specify the domain in interval notation of each one.

   (a) [3] $fg(x)$

   $fg(x) = \sqrt{7x-3}(3x-7)$. The domain is the solution of $7x - 3 \geq 0$: We have $7x \geq 3$, so $x \geq \frac{3}{7}$. The interval notation, we have $\left[\frac{3}{7}, \infty\right)$.

   (b) [4] $\left(\frac{f}{g}\right)(x)$

   $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{7x-3}}{3x-7}$. A real number x is in the domain if x is in the domain of both f and g and $g(x) \neq 0$. So, we have to determine when $7x - 3 \geq 0$ and $3x - 7 \neq 0$. Above, we found $7x - 3 \geq 0$ on $[3/7, \infty)$. Solving $3x - 7 = 0$, we get $x = 7/3$. Thus, we have the domain $[3/7, 7/3) \cup (7/3, \infty)$.

   (c) [4] $\left(\frac{g}{f}\right)(x)$

   $\left(\frac{g}{f}\right)(x) = \frac{3x-7}{\sqrt{7x-3}}$. We have to determine when $7x - 3 > 0$. This occurs on $(3/7, \infty)$, and thus that is the domain.

   (d) [4] $f(g(x))$

   $f(g(x)) = \sqrt{7g(x) - 3} = \sqrt{7(3x-7) - 3} = \sqrt{21x - 52}$. For the domain, we need $g(x)$ to be in the domain of f, in other words, $3x - 7 \geq 3/7$. So $3x \geq 52/7$, that is, $x \geq 52/21$.

10. Simplify:

$$\frac{\sqrt{c^2 d^6}}{\sqrt{4c^3 d^{-4}}} \qquad\qquad \log_2 \frac{1}{4}$$

11. [7] Given $f(3) = 0$, use the factor theorem to find the other roots of $x^4 - 3x^3 - 25x^2 + 75x$

We know $x - 3$ is a factor of f by the factor theorem. Divide f by $x - 3$, to get $x^3 - 25x$. Note we can factor this to get $x(x^2 - 25) = x(x+5)(x-5)$. Thus, the roots are $3, 5, -5, 0$.

12. [3] Simplify:

$$\frac{(x^2)^{\frac{1}{3}}(8y^2)^{\frac{2}{3}}}{4x^{\frac{2}{3}}y^2}$$

$$2 - \log_5(25z)$$

$2 - \log_5(25z) = 2 - (\log_5(25) + \log_5(z)) = 2 - (2 + \log_5(z)) = -\log_5(z)$.

13. [4] Solve for x:

$$\log(x - 16) = 2 - \log(x - 1)$$

$$4^x - 3 * 2^x = 10$$

Note $4 = 2^2$. So, we can write $(2^2)^x - 3 \cdot 2^x = 10$, so $(2^x)^2 - 3 \cdot 2^x = 10$. Let $u = 2^x$. Then we have $u^2 - 3u - 10 = 0$, that is, $(u-5)(u+2) = 0$. So, $u = 5$ or $u = -2$. So $2^x = 5$ or $2^x = -2$. The second is impossible, so we have $2^x = 5$, which means $x = \log_2(5)$.

14. [4] Draw the complete graph of $f(x) = \frac{1}{2}\log_2(x+3)+1$, using only graph transformations. List the transformations in order.

The transformations are

- *Horizontal shift left* 3 *units*
- *Vertically stretch by a factor of* $\frac{1}{2}$
- *Vertical shift up* 1 *unit*

15. [4] Your given a 16 oz mocha that is a rather weak 3% espresso. You, knowing you'll be up late studying mathematics, would rather like a 30% espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso?

Set up the following equation, where x is the amount of the drink to be discarded and replaced:

$$((16-x) \text{ oz. of 3\% espresso}) + (x \text{ oz. of 100\% espresso}) = (16 \text{ oz. of 30\% espresso}).$$

This can be rewritten

$$.03(16-x)+x = 16(.3).$$

We can solve for x to get $x \approx 4.45361$.

16. [2] How long will a loan take to triple at 20% interest compounted quarterly?

The amount of money owed after t years is given by $f(t) = P\left(1+\frac{.2}{4}\right)^{4t}$, where P is the original principal. We want $f(t) = 3P$, that is, $3P = P(1+.2/4)^{4t}$. We can immediately cancel the P's to get $3 = (1+.2/4)^{4t}$, that is, $3 = 1.05^{4t}$. Apply ln *to both sides, obtaining*

$$\ln(3) = \ln(1.05^{4t})$$
$$\ln(3) = 4t\ln(1.05)$$
$$\frac{\ln(3)}{4\ln(1.05)} = t$$
$$t \approx 5.62927$$

. *So, you should leave your money in until third quarter of the fifth year.*

17. [2] A potter can sell 120 bowls per week at \$5 per bowl. For each 50 cent decrease in price, 20 more bowls are sold. What price should be charged to maximize sales income?

18. [10] Graph and answer the following for

$$f(x) = \frac{(x-2)(x^2+3x-10)}{x^2-5x+6}$$

.

(a) [3] Domain: (Give answers in interval notation)

The domain comprises all real numbers that are not roots of the denominator. We can factor to get $f(x) = \frac{(x-2)(x-2)(x+5)}{(x-3)(x-2)}$. *Thus, the domain is* $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$.

(b) [2] Vertical Asymptotes:

Simplify $f(x)$ *to* $g(x) = \frac{(x-2)(x+5)}{(x-3)}$. *The vertical asymptotes are the roots of the denominator of this simplified function. Thus there is a vertical asymptote at* $x = 3$

(c) [1] Horizontal Asymptotes:

The numerator has degree 2*, while the denominator has degree* 1*. Thus, there is no horizontal asymptote.*

(d) [2] X-Intercepts:

We look at roots of the numerator, which occur at $x = 2$ *and* $x = -5$*. However, there is a hole at* $x = 2$*, so the only* x*-intercept is at* $x = -5$*. The coordinates are* $(-5, 0)$.

(e) [1] Y-Intercept:y

Evaluate the function at $x = 0$*:*

$$f(0) = \frac{(-2)5}{(-3)} = 10/3.$$

(f) [2] Holes: (Just the x-coordinate will suffice.)

Examining the domain, we see $x \neq 2$ *and* $x \neq 3$*. From above we know* $x = 3$ *is a vertical asymptote, but* $x = 2$ *isn't, thus there is a hole at* $x = 2$.