Contents

1. Review

1.1 Algebra Review

I generally assume proficiency in the skills listed below.

Basic Simplification

Apply order of operations to numerical and algebraic expressions; recall the rules for exponents.

Ex 1. Perform the operations: $\dfrac{3}{5} + 2$

Solution:

$\dfrac{3}{5} + \dfrac{2}{1} = \dfrac{3}{5} + \dfrac{2}{1} \cdot \dfrac{5}{5} = \dfrac{3}{5} + \dfrac{10}{5} = \dfrac{13}{5}$

(Another answer is 2.4.)

Ex 2. Simplify: $\sqrt{5} \cdot \sqrt{5} + 2$

Solution:

$\sqrt{5} \cdot \sqrt{5} + 2 = 5 + 2 = 7$

Ex 3. Simplify: $\sqrt{\dfrac{4}{25}}$

Solution:

$\sqrt{\dfrac{4}{25}} = \dfrac{\sqrt{4}}{\sqrt{25}} = \dfrac{2}{5}$

Ex 4. Simplify: $-3^2 + \left(\frac{3}{2}\right)^2 + (-2)^3$.

Solution:

$-3^2 + \left(\dfrac{3}{2}\right)^2 + (-2)^3 = -9 + \dfrac{9}{4} + -8 = -17 + \dfrac{9}{4} = \dfrac{-68}{4} + \dfrac{9}{4} = \dfrac{-59}{4}$

(Another answer is -14.75. $-14\frac{3}{4}$ is also technically correct, but you should avoid mixed fractions because of potential confusion: Taken out of context, $-14\frac{3}{4}$ could mean *either* $-(14 + \frac{3}{4})$ *or* $(-14) \cdot \frac{3}{4}$.)

Algebraic Manipulation

Add, subtract, multiply, and divide algebraic expressions; combine algebraic terms that are alike; apply the distributive property to algebraic expressions (the term "FOIL" *only* refers to distribution between two binomials; ask me to show you why FOIL works if you don't remember).

Ex 5 Solve for x: $\dfrac{3}{x} = \dfrac{1}{6}$.

Solution:

Multiply both sides by x: $x \cdot \frac{3}{x} = \frac{1}{6} \cdot x \Rightarrow 3 = \frac{x}{6}$.

Multiply both sides by 6: $6 \cdot 3 = \frac{x}{6} \cdot 6 \Rightarrow 18 = x$.

Both of these steps done at the same time is often called "cross multiplication".

Ex 6 Simplify by combining like terms: $(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1)$.

Solution:

$(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1) = 9x^2 + 4xy - 7y^2 - 5xy + 6x^2 + 1 = 15x^2 - xy - 7y^2 + 1$

Ex 7 Expand and simplify completely: $3(4v - 2a)^2$.

Solution:

$$\begin{aligned} 3(4v - 2a)^2 = 3(4v - 2a) \cdot (4v - 2a) &= 3[(4v)(4v) - (4v)(2a) - (2a)(4v) + (2a)(2a)] \\ &= 3[16v^2 - 16av + 4a^2] = 48v^2 - 48av + 12a^2 \end{aligned}$$

Rational Expressions

Simplify fractions by finding factors in common; add/subtract fractions by first rewriting with the least common denominator; multiply/divide rational expressions; simplify complex fractions.

Ex 9 Reduce to lowest terms: $\frac{10x - 20}{2x^2 - 8}$.

Solution:

$$\frac{10x - 20}{2x^2 - 8} = \frac{10(x - 2)}{2(x^2 - 4)} = \frac{10(x - 2)}{2(x + 2)(x - 2)} = \frac{2(x - 2)}{2(x - 2)} \cdot \frac{5}{x + 2} = \frac{5}{x + 2}$$

Ex 8 Perform the indicated operations and simplify: $\frac{2}{3x + 2} - \frac{1}{3x + 1}$.

Solution:

$$\begin{aligned} \frac{2}{3x + 2} - \frac{1}{3x + 1} &= \frac{3x + 1}{3x + 1} \cdot \frac{2}{3x + 2} - \frac{1}{3x + 1} \cdot \frac{3x + 2}{3x + 2} \\ &= \frac{6x + 2}{(3x + 1)(3x + 2)} - \frac{3x + 2}{(3x + 1)(3x + 2)} = \frac{3x}{(3x + 1)(3x + 2)} \end{aligned}$$

(There is no further simplification here, $3x$ has *no* factors in common with $3x + 1$ or $3x + 2$.)

Ex 9 Perform the indicated operations and simplify: $\frac{2z + 6}{12z} \div \frac{z^2 - 9}{9z^3 + 18z^2}$.

Solution:

$$\begin{aligned} \frac{2z + 6}{12z} \div \frac{z^2 - 9}{9z^3 + 18z^2} = \frac{2z + 6}{12z} \cdot \frac{9z^3 + 18z^2}{z^2 - 9} &= \frac{2(z + 3)}{12z} \frac{9z^2(z + 2)}{(z + 3)(z - 3)} \\ &= \frac{2 \cdot 3 \cdot z(z + 3)}{2 \cdot 3 \cdot z(z + 3)} \cdot \frac{3z(z + 2)}{2(z - 3)} = \frac{3z(z + 2)}{2(z - 3)} \end{aligned}$$

Solving Equations

Solve linear equations; or solve for two unknowns when given two equations.

Ex 10 Solve for m: $3(m + 4) + 2m = 4 - 3m$.

Solution

$3(m + 4) + 2m = 4 - 3m \Rightarrow 3m + 12 + 2m = 4 - 3m \Rightarrow 8m = -8 \Rightarrow m = -1$

Ex 11 Solve for q when $q + 7 = 9p$ and $\frac{2}{p} = 2q$.

2. Patty Paper Activities

Welcome to geometric investigations with patty paper!

Patty paper is a thin wax paper commonly used between uncooked hamburger patties. They are conveniently cut into 5.5" or 6" squares which is useful to have during origami practice or for geometric investigations.

Patty Paper Rules

Just as video games, car drivers, and people in general have rules that must be respected, our geometric investigations are going to have a set of rules. A few years ago most geometry classes would only let students use a compass and straightedge (no rulers or protractors!). The calculus classes on campus will let students use rulers, protractors, and calculators but no graphing calculators. Every class and in particular, every math class, is usually explicit about what tools the students are allowed to use in order to solve problems, that is, they lay out the 'rules'.

In this class we will have the patty paper game rules:
You are allowed to use:

1. lots of patty paper (notice that they are semi-translucent and stackable!),
2. the assumption that the patty paper is a square,
3. a pencil (or pen, colored pencil, crayons, etc),
4. a calculator (of any kind),
5. your senses (seeing, hearing, feeling, etc), and
6. logic.

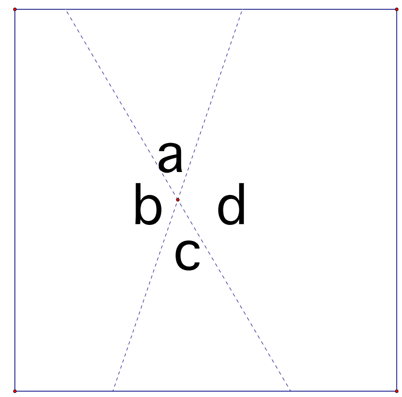
Explicitly that means you are *not* allowed to use: an already made ruler or an already made protractor.

Despite these restrictions you will be able to complete many tasks. For example, while distances can't be measured, we can make a record of it by making marks on the paper. With this record of distance we can compare distances to each other to determine which is longer or if they are the same length. As the investigations go on you will discover your own methods and should build your own 'tool set' of techniques.

The activities are designed to be started in class and finished as part of your homework. These problems are intentionally non-standard so that you can practice scientific thinking when working on them (SLO content #2, skills #4 &6) and then develop technical communication skills when writing your solutions (SLO content #3, skills #1). Your finished worksheet should be easy to read with complete sentences, correct grammar, and precise language. You *will* need to reread, edit, and rewrite your solutions and should *not* be turning in a first draft.

2.1 Patty Paper Activity 1

Vertical Angles



inspired by Michael Serra's *Patty Paper Geometry*.

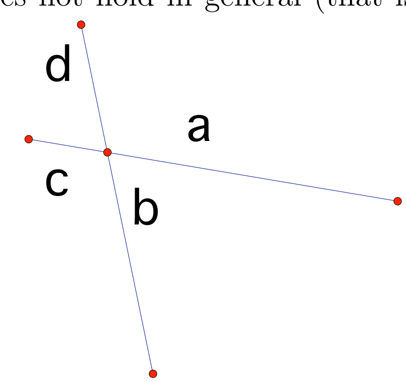
1. Fold a line on a patty paper. Unfold.
   Fold a second line intersecting the first line. Unfold.

2. Notice there are four angles surrounding the intersection of the two lines. Label them a, b, c, and d so that you can refer to each by name.

3. Are any angles the same? Are all the angles different?

4. Explain *carefully* what steps you took so that you could compare one angle to another. Consider drawing pictures if that helps clarify your steps.

5. Repeat steps 1 and 2 and see if the observations you made in Step 3 still apply to a different pair of lines. If your conclusion in Step 3, does not hold in general (that is for any pair of intersecting lines that make angles a, b, c, and d), correct Step 3 so that it is true in general.

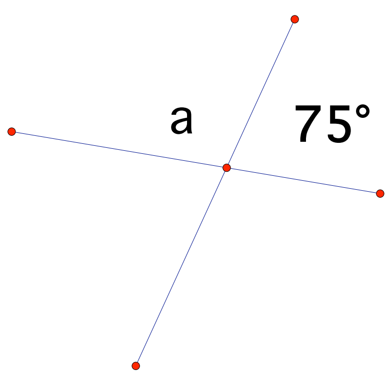


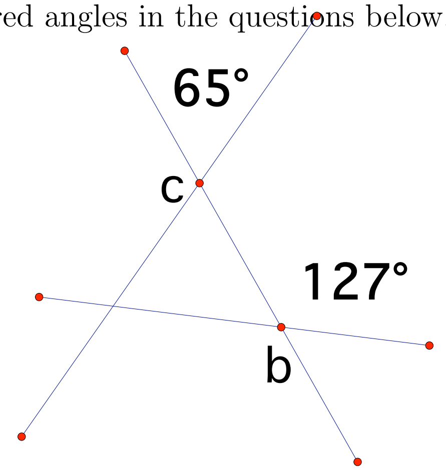
6. The pairs of opposite angles formed by two intersecting lines are called *vertical angles*. For example, in the diagram to the right, $\angle a$ and $\angle c$ are a pair of vertical angles. Restate your conclusions in 3 using this new terminology.

7. Two angles that have a common vertex, share a side, and do not overlap are *adjacent angles*. For example, $\angle a$ and $\angle b$ in the diagram on the bottom of the front page, are adjacent angles. Can you say anything about sum of the pairs of adjacent angles above?

8. Two angles whose measures add up to 180° are called *supplementary* angles. Are all adjacent angles also supplementary? Either justify yourself of provide a counter example.

Use your conclusions on the front page of this worksheet and any other geometric knowledge to calculate the measure of each lettered angles in the questions below.





9. Origami directions usually start with a square piece of paper, but 8.5"×11" paper is more commonly available today. How can you determine what amount to cut off of a rectangular 8.5"×11" (remember the rules! no rulers!) to make a square? Give the steps below (you may want another sheet of paper if you need more room) and then *justify* that your steps guarantee a square.

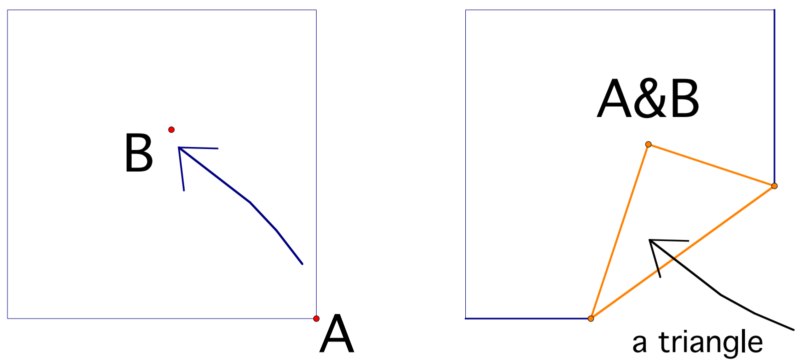
2.2 Patty Paper Activity 2

Folding TUPs

inspired by Kazuo Haga's "Folding Paper and Enjoy Math: Origamics" in *Origami: Third International Meeting of Origami Science, Mathematics, and Education.*

1. Take a piece of patty paper and label the lower right-hand corner A. Pick a random point on the paper and label that point B.

2. Fold the paper so that A lies on top of B. This creates a flap of paper, called the Turned-Up Part (or TUP for short).



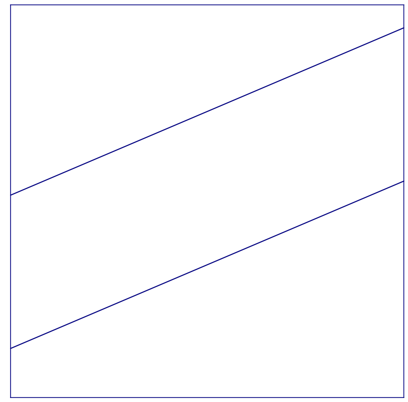
3. [1] How many edges does your TUP have? Three? Four? Five?

   For instance, in the example depicted above, the TUP has three sides.

4. [7] Experiment with many TUPs to find an answer to the question, "Given a point B on the patty paper, how can we tell how many edges a TUP will have before we even fold the paper?"

5. [7] What if we allowed the point B to be outside the square? "Given a point B (plotted anywhere!), how can we tell how many edges a TUP will have before we even fold the paper?"

2.3 Patty Paper Activity 3

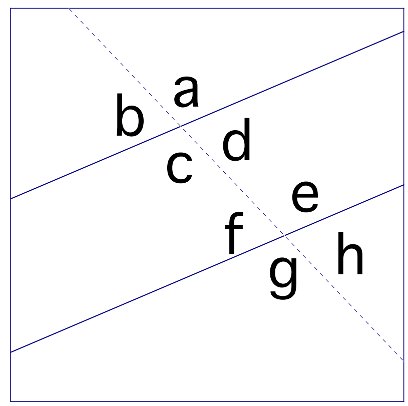


Angles & Parallel Lines

inspired by Michael Serra's *Patty Paper Geometry.*

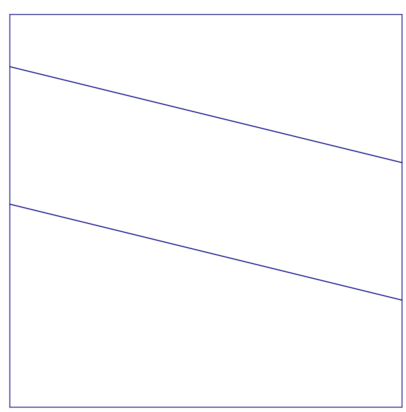
1. Trace the pair of parallel lines shown on the right, onto a patty paper.

2. Fold or draw a transversal line.
   Label the angles as shown in the diagram
   so that you can refer to each by name.



3. [1] Recall that we started with two parallel lines.
   Do any angles have the same measure? Which ones?
   (Write a complete list.)

4. [5] Explain *carefully* what steps you took so that you could compare one angle to another. Consider drawing pictures if that helps clarify your steps.



5. Repeat steps 1 through 3 with a different pair of parallel lines (another set is provided on the right if you are not sure how to manufacture your own) and see if the observations you make in Step 3 still apply. If your conclusion in Step 3, does not hold in general (that is for any set of parallel lines intersected by a transversal), correct Step 3 so that it is true in general.

6. [2] Recall the definition of corresponding angles provided in class and that Step #3 assumed the two given lines were parallel. Restate and, if necessary, expand your statement in Step 3 so that you use this term.

7. [5] Assume you are given a set of lines with a transversal. If the corresponding angles have the same measure, explain why the alternating interior angles must have the same measure.

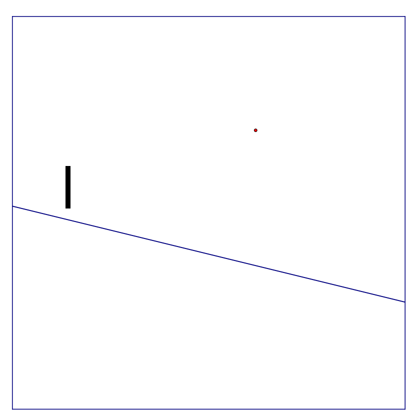
8. [2] Recall that Step #3 assumed the two given lines were parallel, restate and, if necessary, expand your statement in Step 3 that that you use the terms alternating interior angles and alternating exterior angles.

2.4 Patty Paper Activity 4

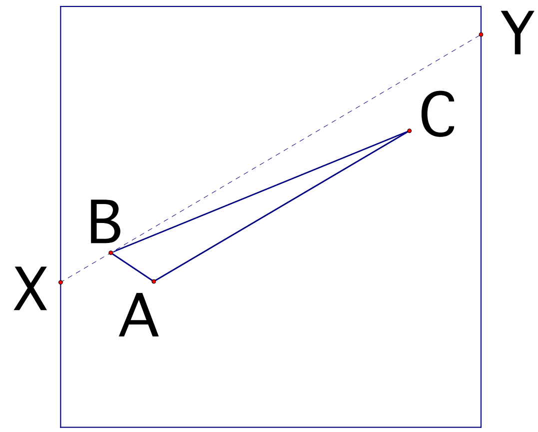
Making Parallel & Perpendicular Lines

inspired by Michael Serra's *Patty Paper Geometry*.

1. Fold a line on a patty paper. Unfold. Mark this line l.
2. Discover a method for making a line that is perpendicular to l.
   Remember that this is a patty paper investigation so stick to the patty paper rules!
3. Make another line on your patty paper and follow your process developed in Step 2 to make a perpendicular line to this new one. If the process you developed in Step 2 does not work, create a new process that will work for *any* line on the patty paper.
4. [4] Discover a method for making a line that is parallel to l. Make sure that your process works for *any* line on your patty paper, and then describe your process. *Justify* why your method works.
5. Use your pencil to make a point not on the line l. Discover a method for folding a line through the point so that it is parallel to the line l. Compare your method with other groups and, if different, determine which method you like better.
6. [5] Describe your favorite process of making a line parallel to a given line l that also runs through a specified point.



Sum of Angles in a Triangle



1. Draw a triangle on your patty paper near the center and identify the vertices A, B, and C.

2. Use the method described on the front of this worksheet to make a line parallel to AC that also passes through B. Label the points where this new line intersects with the edges of the patty paper as X and Y, as shown.

3. [1] There should be at least five angles on your patty paper now. Are any of the angles on the patty paper are the same? If so, mark them.

4. Repeat Steps 1 and 2 with a different triangle to see if the observations you made in Step 3 still apply. If your statements in Step 3 does not hold in general (for any $\triangle ABC$ and line parallel to $\overline{AC}$), correct Step 3.

5. [2] The $\angle XBY$ is a straight line and thus measures $180°$, but it is also equal to the sum three other angles. What three angles make up $\angle XBY$?

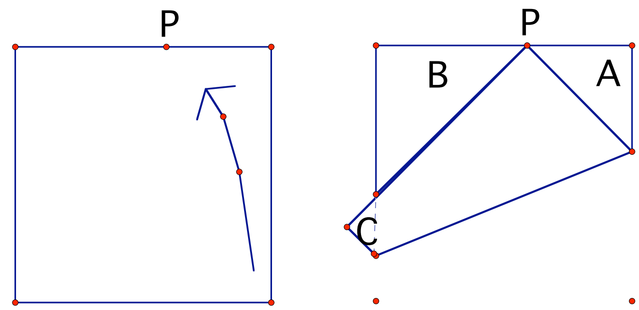
6. [3] Use Step 5 and your observations made in Step 3 to *justify* that the sum of angles in a triangle is $180°$.

2.5 Patty Paper Activity 5

A Triangle Theorem

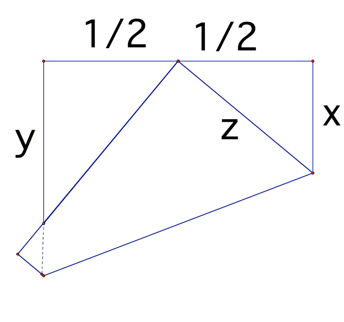
inspired by Kazuo Haga's "Folding Paper and Enjoy Math: Origamics" in *Origami: Third International Meeting of Origami Science, Mathematics, and Education.*

1. Take a patty paper and mark a point P at random along the top edge of the paper.
2. Fold the lower right corner to the point P as depicted.



3. Notice that there are (usually) three triangles that are formed by the fold you made in step two. Label the triangles A, B, and C as done above.
4. [7] What nice relationship must be true about triangles A, B, and C? Justify your conclusions.

5. [8] Suppose that you took the point P to be the midpoint of the top edge. Use your above observation (or generate more observations) to find out what lengths x, y, and z must be in the figure to the right.



2.6 Patty Paper Activity 6

The Largest Equilateral Triangle

inspired by Thomas Hull's "What's the biggest Equilateral Triangle in a Square" in *Project Origami, activities for exploring mathematics.*

1. [5] First fold you square to produce a $30° - 60° - 90°$ triangle inside it. Explain your method and *justify* that you made a $30° - 60° - 90°$ triangle . Hint: these special triangles have the property that the hypothenuse is twice as long as one of its sides.

2. [5] Find a way to fold an equilateral triangle inside a patty paper. Explain your method and *justify* that you made an equilateral triangle.

3. [1] If the side length of your original square is 1, conjecture if it is possible for you to make an equilateral triangle who's side length is greater than 1?

4. [3] If an equilateral triangle is maximal (the equilateral triangle is the largest equilateral triangle that will fit on a patty paper), can we assume that one of its vertices will coincide with a vertex of the patty paper? Why? Could we say anything about the other vertices of the equilateral triangle?

5. [1] Draw a picture of the situation in question 4 where the patty paper shares the lower left vertex with the maximal equilateral triangle.

6. [5] Let θ denote the angle between the bottom of the square and the bottom of the triangle. What angle should θ be to maximize the size of the equilateral triangle in the patty paper? *Justify* your conclusion. Hint: consider symmetry!

2.7 Patty Paper Activity 7

Spherical Geometry

In addition to the normal patty paper rules, assume that you have a perfect sphere.

1. Explain in your own words what a geodesic is.

2. Answer each of the following:

   How many geodesic paths are between two points in a plane?

   How many geodesic paths are between the 'north pole' & 'south pole' on a sphere?

3. [3] You have already come up with methods to make a geodesic perpendicular to a given one on a plane (think Worksheet 4). Find a (patty paper) technique to make a geodesic perpendicular a given one on the sphere. Explain your steps and *justify* how you know your constructed geodesic is perpendicular.

4. [2] You have already come up with methods to make a geodesic parallel to a given one on a plane (think Worksheet 4 again). Can you find a technique to make a geodesic parallel to a given one on the sphere? *Justify* your answer.

5. [2] Answer each of the following:

How many times do two perpendicular geodesics intersect in the plane?

How many times to two perpendicular geodesics intersect on the sphere?

If two geodesics are perpendicular on a plane, how many 90° angles are made?

If two geodesics are perpendicular on a sphere, how many 90° angles are made?

Recall that I defined a *triangle* in class as figure with three sides. On a plane we understand 'side' to mean line, but really, we should think of 'side' as a geodesic. Thus, a *triangle* is a figure made from three geodesics.

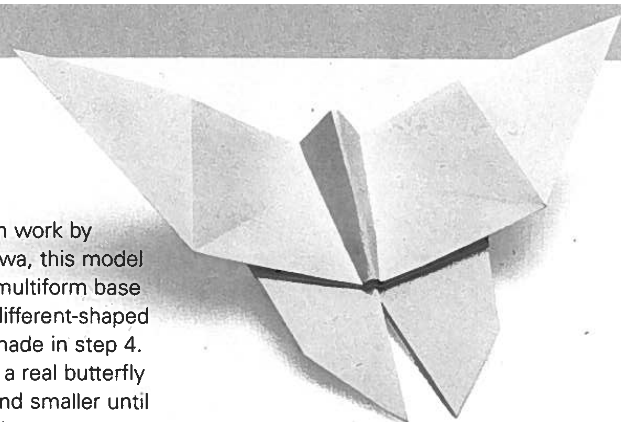
6. [4] Can you make a right triangle on the sphere? If so, explain how, and if not, explain why you can't.

7. [2] Can you make a right triangle on the sphere with more than one 90° angle? If so, draw a picture of it, and if not, explain why you can't

8. [2] Can you make a rectangle on the sphere? If so, draw a picture of it, and if not, explain why you can't

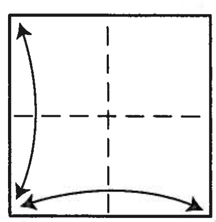
3. Origami Patterns

3.1 Butterfly

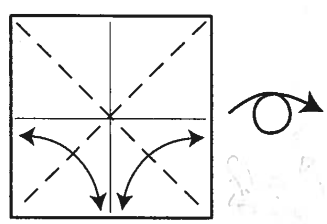


A design that probably started with work by the Japanese master Akira Yoshizawa, this model uses the folding sequence of the multiform base to form the wings. You can make different-shaped lower wings by altering the folds made in step 4. Try to choose paper that looks like a real butterfly and see if you can fold it smaller and smaller until it is the same size as a real butterfly.

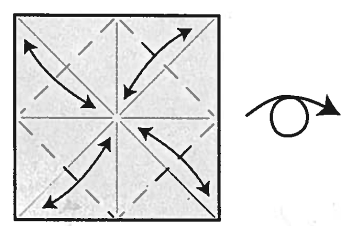
MULTIFORM BASE REMINDER



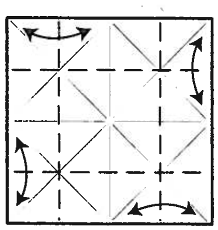
1 Start with a square, white side up. Fold in half from side to side and top to bottom, crease, then unfold.



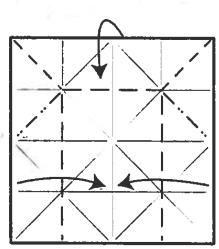
2 Crease both diagonals.



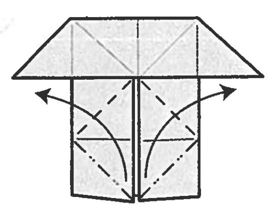
3 Turn the paper over and fold each corner to the center. Crease and unfold. Turn the paper back over.



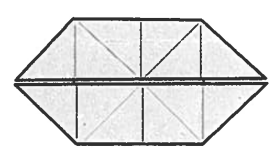
4 Fold each edge to the center. Crease and unfold.



5 Use the creases indicated to collapse the paper—no new creases are needed.



6 Collapse the lower section to match the top.

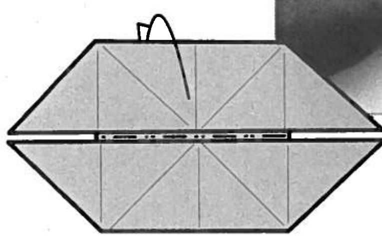


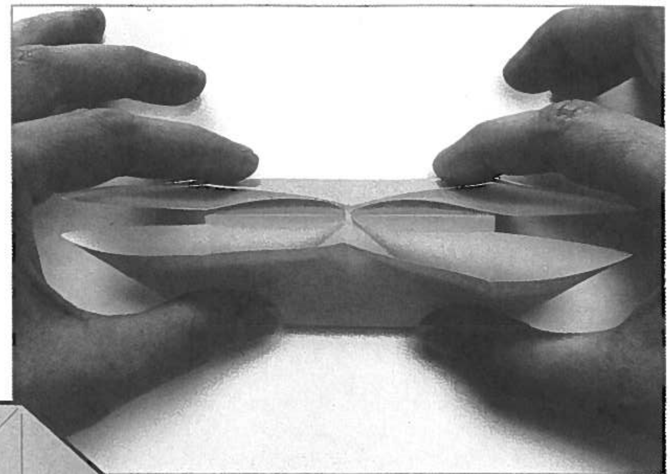
7 The completed multiform base (see page 31).

MAKING THE BUTTERFLY

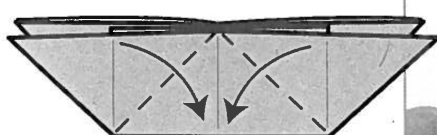
1 Start with a multiform base (see opposite).

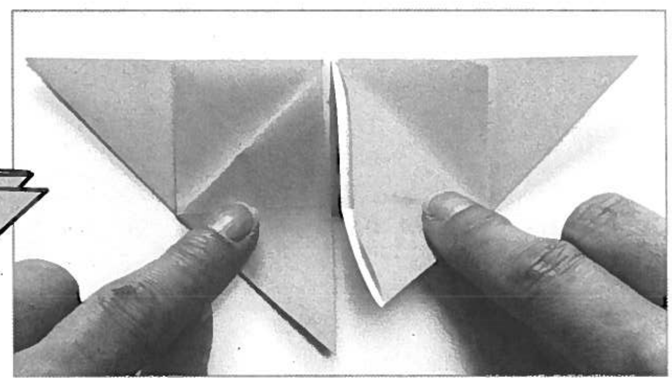
▼ **2** Mountain fold (see page 15) in half.



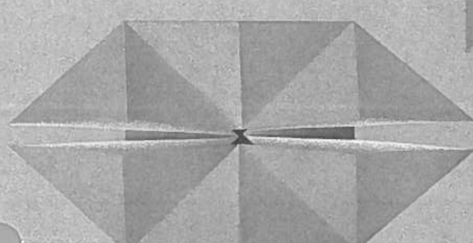


▼ **3** Swing the two flaps downward.



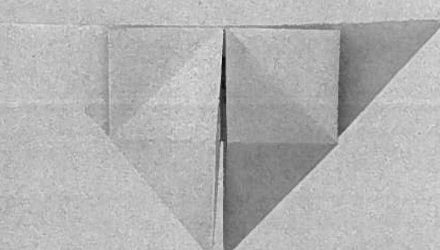


→ WATCH THE MODEL FOLD UP...

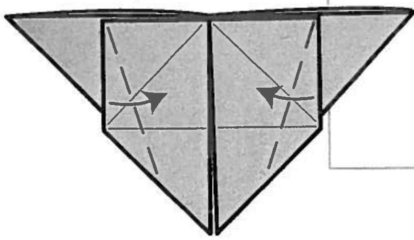


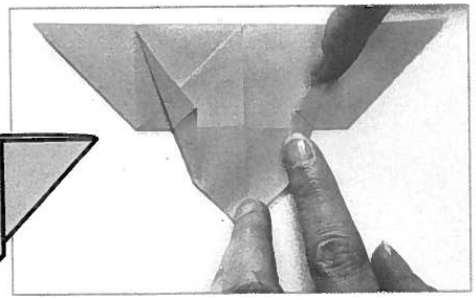
1 The multiform base

2 Fold in half



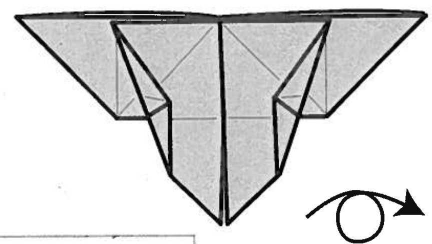
3 Swing two flaps down

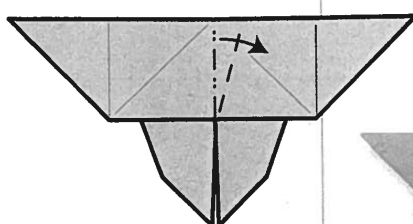




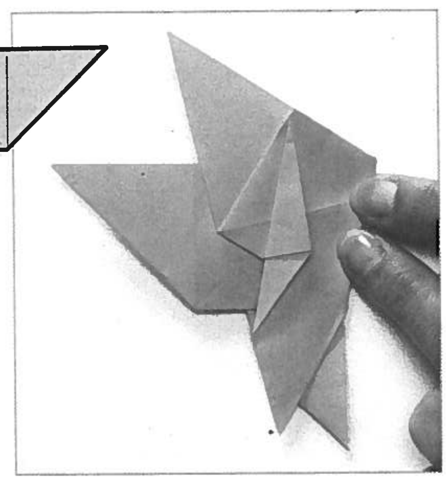
▲ **4** Shape the wings with two new folds starting at the right-angled corners, at a slight angle to the vertical.

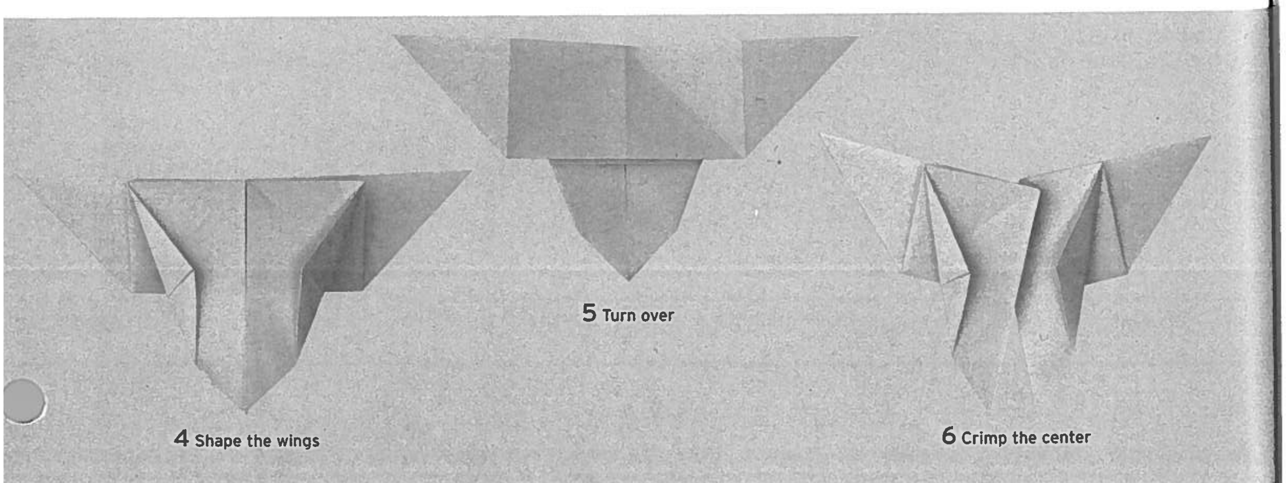
► **5** This is the result. Turn the paper over.

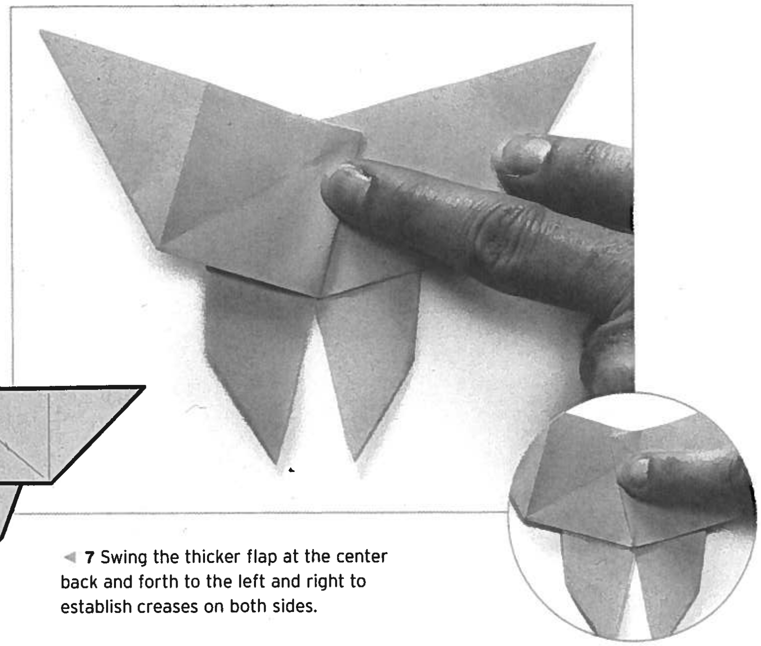


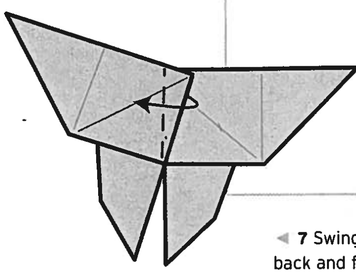


▲ **6** Make a small crimp (see page 22) so the wings are at a slight angle.

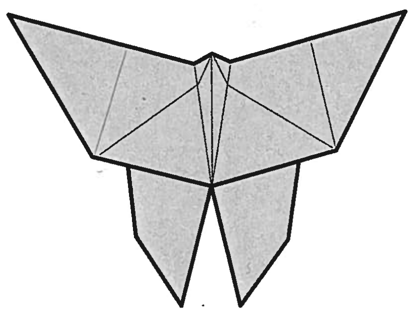




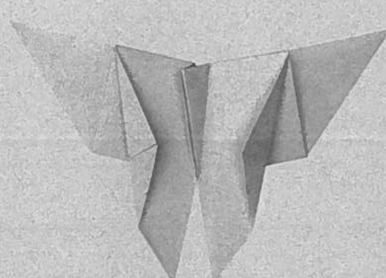




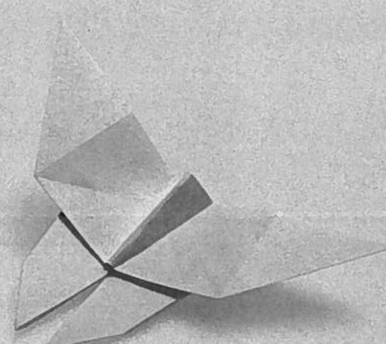
◄ **7** Swing the thicker flap at the center back and forth to the left and right to establish creases on both sides.



► **8** Open the wings so that the center is raised for the finished butterfly.



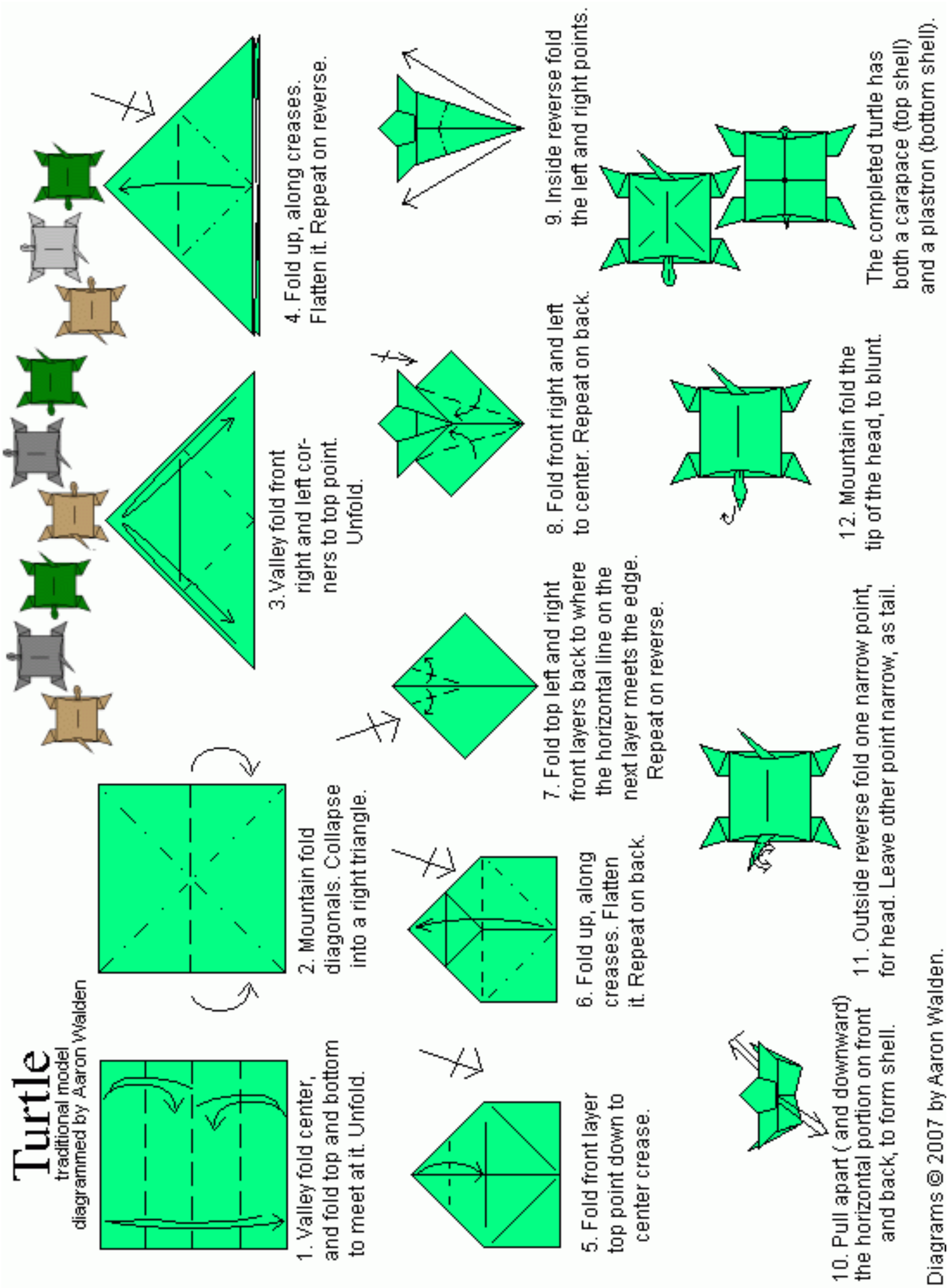
7 Swing the crimp back and forth to crease it on both sides



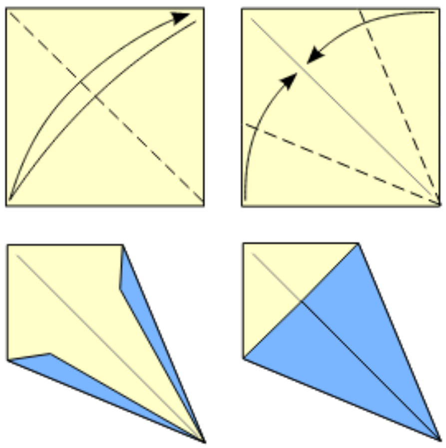
8 Open the wings and raise the center to complete the butterfly

Robinson Nick. 2008. Picture-perfect Origami. New York: St. Martin's Griffin.

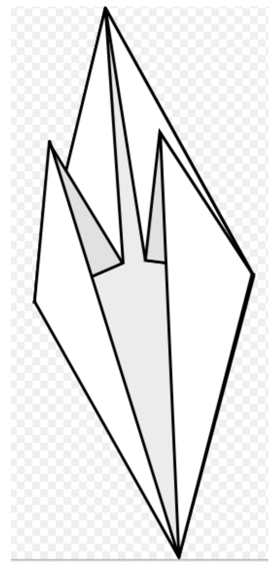
3.2 Turtle


Turtle
traditional model
diagrammed by Aaron Walden
1. Valley fold center, and fold top and bottom to meet at it. Unfold.
2. Mountain fold diagonals. Collapse into a right triangle.
3. Valley fold front right and left corners to top point. Unfold.
4. Fold up, along creases. Flatten it. Repeat on reverse.
5. Fold front layer top point down to center crease.
6. Fold up, along creases. Flatten it. Repeat on back.
7. Fold top left and right front layers back to where the horizontal line on the next layer meets the edge. Repeat on reverse.
8. Fold front right and left to center. Repeat on back.
9. Inside reverse fold the left and right points.
10. Pull apart (and downward) the horizontal portion on front and back, to form shell.
11. Outside reverse fold one narrow point, for head. Leave other point narrow, as tail.
12. Mountain fold the tip of the head, to blunt.
The completed turtle has both a carapace (top shell) and a plastron (bottom shell).
Diagrams © 2007 by Aaron Walden.

3.3 Bases (classic)

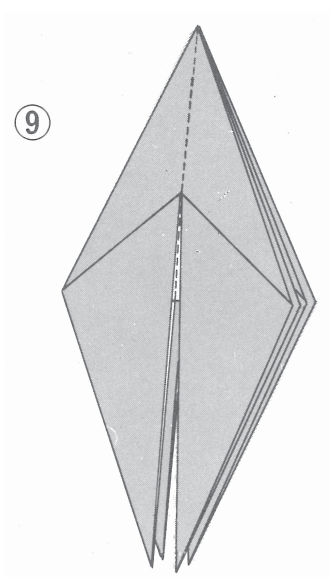


Kite Base



Fish Base

Made available through Wikibooks.org under the wikimedia commons or GNU free documentation.

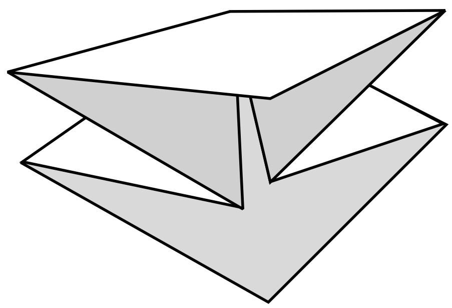


Frog Base

From: Honda, Isao.1965. The World of Origami. Tokyo: Japan Publications Trading Company.

3.4 Bases (other)

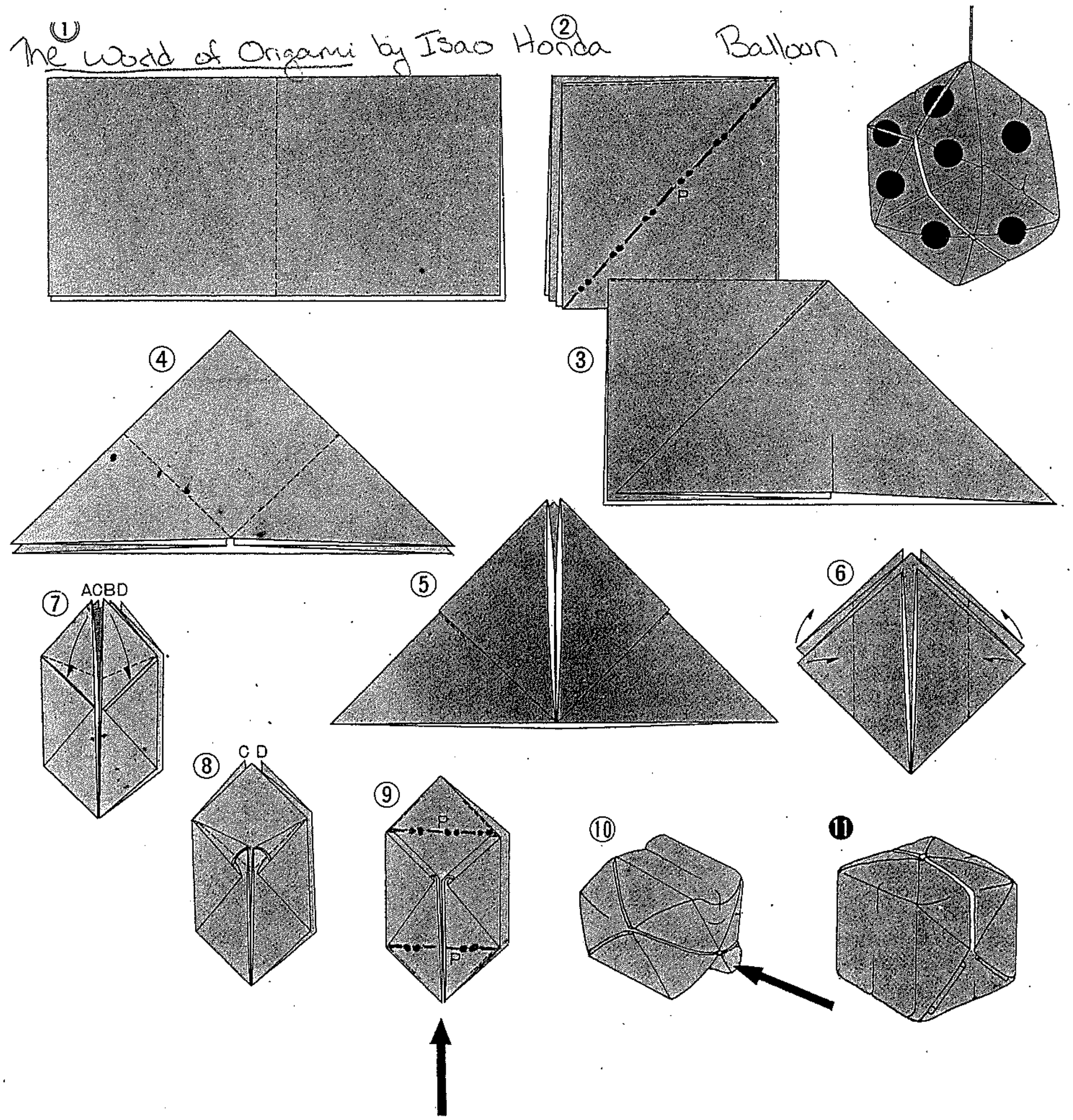
4/8/13 9:24 AM



Preliminary Fold
Available under the wikimedia commons through Wikibooks.org.

file:///Users/ruth/Desktop/PreliminaryFold.svg Page 1 of 1

3.5 Balloon



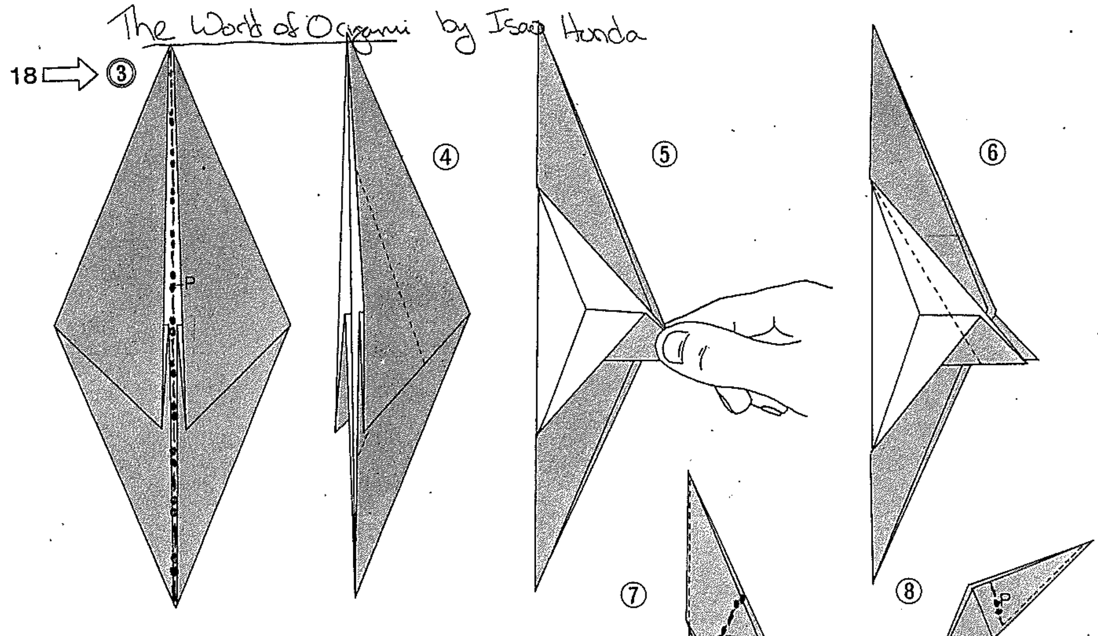
72. Balloon

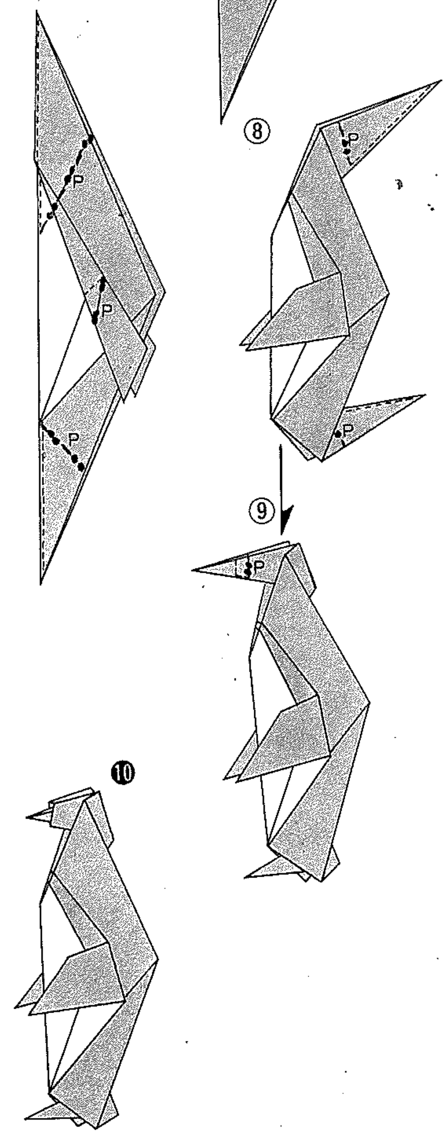
The balloon base is an origami that Japanese children learn in kindergarten. With it we can make a number of interesting folds.
Use a six-inch square of paper.

1. Prepare the paper by book folding it in half. Valley fold on the dotted line.
2. Open the square into a triangle by peak and valley folding as indicated.
3. Repeat step 2 on the underside.
4. Valley fold the top folds on the dotted lines, bringing the bottom edges up to the center line.
5. Turn the model over and repeat.
6. Valley fold the right and left points into the center line. Turn the model over and repeat.
7. Valley fold points A and B down into the pockets shown.
8. Repeat with points C and D.
9. Peak and valley fold as shown to flatten out the top and bottom of the cube shape.
10. Blow air into the indicated hole.
11. The finished balloon.

3.6 Penguin

The World of Origami by Isao Honda





19. Penguin (I)

Use a five-inch sheet of black paper with a white underside.

1~3. Begin with the [redacted] base. Peak fold on the dotted line.
4. Valley fold on the dotted line (top two layers only). Be careful to note that there is a dotted line extending beneath the first fold. Turn the model over and repeat.
5. Pull the flipper sections out to reveal the penguin's white belly.
6. Valley fold each flipper on the dotted line.
7. Peak and valley fold the neck section, and reverse fold it outside in. Peak and valley the foot section, and reverse fold it outside in. Peak and valley fold the flippers on the dotted lines.
8. Peak and valley fold the neck on the dotted lines, and reverse fold it outside in to form the head. Peak and valley fold the feet section, and reverse fold it outside in.
9. Peak and valley fold the head, and reverse fold it in to form the beak.

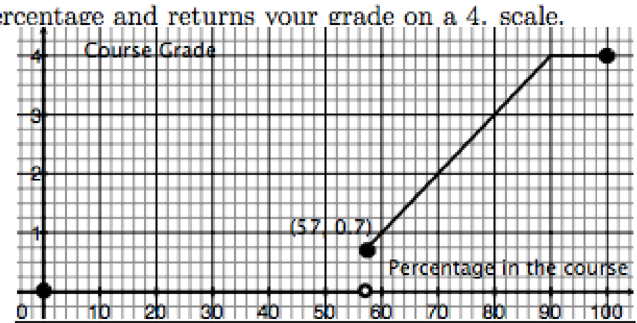
4. Other Activities

4.1 Computing Grades

1. Clark Kent is handed the following information on the first day of Origami Math class:

Grades: The following weights will be used to calculate your percentage in the course. The function graphed takes your course percentage and returns your grade on a 4. scale.

Journal	10%
Homework	20%
Quizzes	15%
Paper & Project	20%
Scaffolding	10%
Midterm	10%
Final	15%



Outside Resources:

In week 7 Clark is concerned about his grades (being a super hero is still new to him so he has missed a few assignments and a quiz). He logs onto Canvas and finds::

Journal:	(ea. worth 5pts)	5	5	5	5	5	5	5	5	5	5	5	5	5
Homework:	(ea. worth 20pts)	15	18	17	0	0								
Quizzes:	(ea. worth 20pts)	20	20	0	0									
Paper/Project:	(ea. worth 100pts)	82	80											
Scaffolding	(ea. worth 5pts)	5	5	5										
Midterm	(ea. worth 100pts)	82												
Final														

(a) Compute Clark's current averages for each of the categories below:

Journal
Homework
Quizzes

Paper/Project

Scaffolding

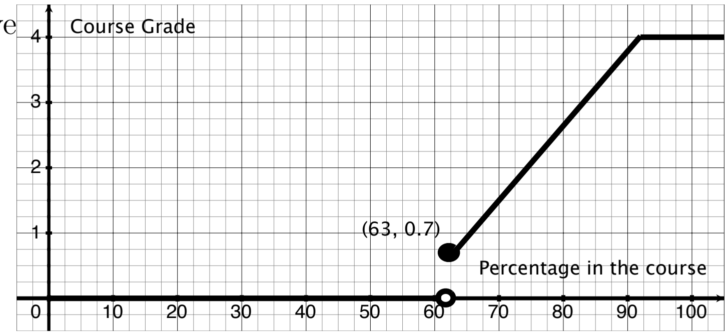
(b) Review the syllabus and see if and marks are dropped. If so, recompute the averages without the dropped score.

(c) Assuming his performance in each category will not change in the remaining weeks, what grade must Clark get on his final to get a 2.0 in the class?

2. [5] James T. Kirk is in TMath 120 and would like to know if it is still possible to earn a 2.0 now that he's taken two exams. He has looked at the grade book on MyMathLab and has computed the averages listed below.

   Assuming James' work does not drastically change in the remaining 3 weeks and his averages remain about the same, find what grade he needs to get on the final to receive a 2.0 in the course. In case you don't remember, the weights specified in the syllabus and the graph of the function f that takes your class percentage x and returns your score on a 4. scale are also provided. *Clearly* show you work!

	weight	James' ave
Mini-Quizzes	5%	95%
WebAssign	10%	50%
WrittenHW	15%	0%
Quizzes	15%	70%
2 Exams	30%	100%
Final	25%	



3. [10] *Clearly* show your work, and use your own scores from this core class to:

   (a) Compute your current averages for each of the categories below (dropping the lowest marks when appropriate):

   Journal
   Homework
   Quizzes

   Paper/Project

   Scaffolding

   (b) Assume your performance in each category will not change in the remaining weeks, what grade must you get on your final to get a 2.0 in the class?

   (c) Consult Canvas' Gradebook and use the summary numbers given there to find what grade you need to get on your final to get a 2.0 in the class. If you have difficulty or get different numbers explain why.

5. Readings

5.1 Quotes

Geometry, throughout the 17^{th} and 18^{th} centuries, remained, in the war against empiricism, an impregnable fortress of the idealists. Those who held ... that certain knowledge, independent of experience, was possible about the real world, had only to point to Geometry: none but a madman, they said would throw doubt on its validity, and none but a fool would deny its objective reference.

-Bertrand Russell

I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world.

-G.H. Hardy

From this proposition it will follow, when arithmetical addition has been defined, that

$$1 + 1 = 2.$$

-Principia Mathematica
A.N. Whitehead & B. Russell

Euclidean Axioms:

1. We can draw a straight line from any point to any other point.
2. We can extend a finite straight line continuously into a straight line.
3. We can completely describe a circle by specifying the center and distance [radius].
4. All right angles are equal to one another.
5. Given a point not on a straight line l, at most one line can be drawn through this point that never meets l.

-Elements
Euclid

[Euclidean geometry] was rigorous and definite. Sure theorems about lines and triangles, circles and squares, following with unimpeachable logic from clearly stated assumptions ... [it] remained unchanged for more than a hundred years.

-Pi in the Sky
John Barrow

Origami Axioms:

1. Given two points p_1 and p_2, there is a unique fold that passes through both p_1 and p_2.
2. Given two points p_1 and p_2, there is a unique fold that places p_1 onto p_2.
3. Given two lines l_1 and l_2, there is a fold that places l_1 onto l_2.
4. Given a point p_1 and a line l_1, there is a unique fold perpendicular to l_1 that passes through point p_1.
5. Given two points p_1 and p_2 and a line l_1, there is a fold that places p_1 onto l_1 and passes through p_2.
6. Given two points p_1 and p_2 and two lines l_1 and l_2, there is a fold that places p_1 onto l_1 and p_2 onto l_2.
7. Given one point p and two lines l_1 and l_2, there is a fold that places p onto l_1 and is perpendicular to l_2.

5.2 Technical Communication

Technical Communication Today

THIRD EDITION

Richard Johnson-Sheehan
Purdue University

Longman

New York San Francisco Boston
London Toronto Sydney Tokyo Singapore Madrid
Mexico City Munich Paris Cape Town Hong Kong Montreal

Technical Communication: Actions, Words, Images

If you are a college student, much of your writing experience until now has probably been *expository* in nature. The root word for expository is "exposition," meaning you have been taught to *display* or *exhibit* information. Primarily, you have been taught how to demonstrate that you have acquired and retained knowledge on specific subjects. Your readers have mostly been teachers and professors.

Technical communication is different from expository writing, because it puts more focus on taking action with words and images. It focuses more on achieving a specific *purpose* with language. It also puts a much heavier emphasis on anticipating the needs of the readers and communicating information clearly and persuasively. Figure 1.2 shows some of the characteristics of technical communication that can set it apart from expository writing.

Fortunately, the skills you learned for expository writing are all adaptable to technical communication. The main difference is that technical communication puts a much greater emphasis on achieving a specific purpose with words and images.

The Qualities of Technical Communication

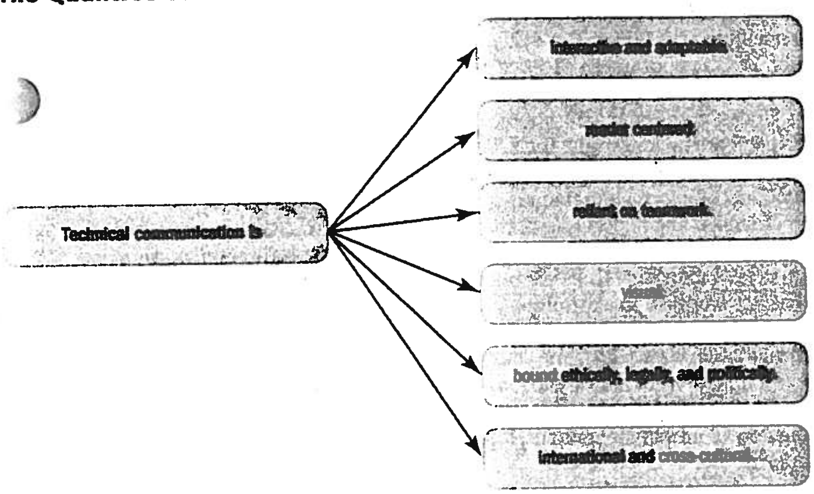


Figure 1.2: Technical communicatic puts much mo emphasis on managing information a taking action do most othe forms of writi

Pg 6

Genres

Genres do much more than help you organize your ideas. They help you interpret and make sense of what is happening in the technical workplace. For example, if you know you need to write an analytical report, the genre will help you figure out what kind of information you need to collect, how that information should be arranged, and how it should be presented. Your readers, meanwhile, will interpret your ideas through the genre. If you call something a "report," they will have specific expectations about the content, organization, style, design, and medium of the document.

Genres are not formulas or recipes to be followed mechanically. They are not containers into which we pour some sentences and paragraphs. Instead, genres in technical communication reflect the activities and practices of scientific and technical workplaces. Each genre should be adapted to fit the readers and the situations in which the document will be used.

Pg 19

Pg 19 and 20

Genres and the Technical Writing Process

A consistent *writing process* is important to using technical communication genres effectively. When learning to write for the workplace, you should view writing as a process that follows a series of stages (Figure 2.2). These stages include the following:

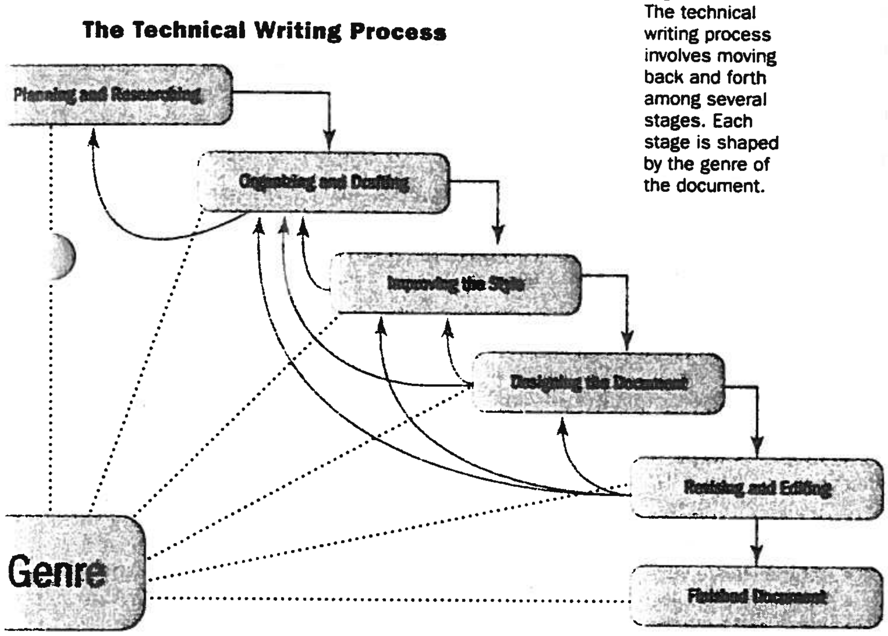


Figure 2.2: The technical writing process involves moving back and forth among several stages. Each stage is shaped by the genre of the document.

Genres in Technical Communication

In this book, you will be learning a *genre-based approach* to the writing process. In technical communication, genres are relatively stable patterns that reflect the activities and practices of the workplace. A genre shapes a document's content, organization, style, design, and the medium in which it is delivered. It also helps you anticipate the needs of your readers and the situations in which they will use your text.

For example, an "analytical report" is a different genre than a "specification" (Figure 2.1). Analytical reports and specifications are written for different kinds of readers and situations. They include different information and follow their own organizational patterns. The style and design of these two genres are distinctly different. Yet, someone working in a technical workplace would need to know how to use both of these genres.

Figure 2.1: Each genre has its own content, organization, style, and design. Here are the outlines of two distinctly different genres set side by side.

More than likely, you have read and used countless sets of instructions in your lifetime. Instructions are packaged with the products we buy, such as phones, cameras, and televisions. In the technical workplace, documentation helps people complete simple and complex tasks, such as downloading software, building an airplane engine, drawing blood from a patient, and assembling a computer motherboard.

Instructions and other kinds of documentation are among the least noticed but most important documents in the technical workplace. Three types of documentation are commonly written and used in this arena:

> **Instructions**—Instructions describe how to perform a specific task. They typically describe how to assemble a product or do something step-by-step.
>
> **Specifications**—Engineers and technicians write specifications (often called the "specs") to describe in exact detail how a product is assembled or how a routine process is completed.
>
> **Procedures/Protocols**—Procedures and protocols are written to ensure consistency and quality in a workplace. In hospitals, for example, doctors and nurses are often asked to write procedures that describe how to handle emergency situations or care for a specific injury or illness. Similarly, scientists will use protocols to ensure consistent methods in the laboratory.

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Link

For more information on writing descriptions, go to Chapter 19, page 530.

To avoid confusion in this chapter, the word ***documentation*** will be used as general term to mean instructions, procedures, and specifications. When the chapter discusses issues that are specific to instructions, procedures, or specifications, those terms will be used.

Basic Features of Documentation

Documentation tends to follow a consistent step-by-step pattern, whether you are describing how to make coffee or how to assemble an automobile engine (Figure 20.1).

Here are the basic features of most forms of documentation:

- specific and precise title
- introduction with background information
- list of materials, parts, tools, and conditions required

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Basic Organization

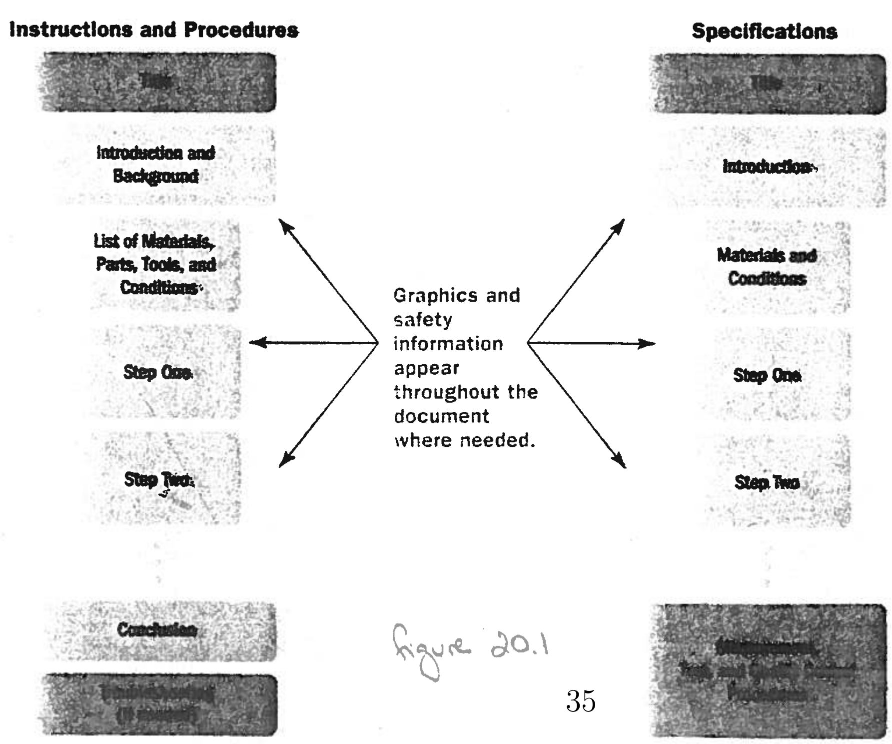
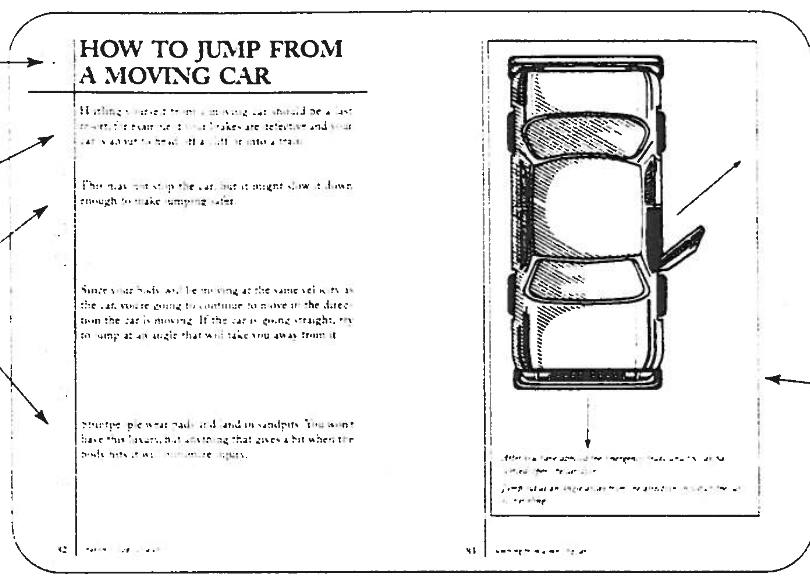


figure 20.1

- sequentially ordered steps
- graphics
- safety information
- measurement, test, and quality control procedures (for specifications)
- conclusion that signals the completion of the task

The content, style, and design of your documentation will change to suit the readers and the contexts in which the text will be used. For example, Figure 20.2 shows survival instructions from an entertaining and useful book called *Worst-Case Scenarios*. These instructions use simple text and visuals to explain how to jump out of a moving car—in case you ever need to.

A Set of Instructions



Graphic visually reinforces steps.

Figure 20.2: Here is a rather simple set of instructions. Notice how the steps make up the bulk of the document.

Source: From the Worst-Case Scenario Survival Handbook™ *by Joshua Piven and David Borgenicht. Copyright © 1999 by Quirk Productions, Inc. Used with permission of Chronicle Books LLC, San Francisco. Visit ChronicleBooks.com.*

Sequentially Ordered Steps

The steps are the centerpiece of any form of documentation, and they will usually make up the bulk of the text. These steps need to be presented logically and concisely, allowing readers to easily understand them and complete the task.

As you carve the task you are describing into steps, you might use logical mapping to sort out the major and minor steps (Figure 20.8). First, put the overall task you are describing on the left-hand side of the screen or a sheet of paper. Then, break the task down into its major and minor steps. You might also, as shown in Figure 20.8, make note of any necessary hazard statements or additional comments that might be included.

Once you have organized the task into major and minor steps, you are ready to draft your instructions.

Use command voice. Steps should be written in *command voice*, or imperative mood. To use command voice, start each step with an action verb.

1. Place the telescope in an upright position on a flat surface.
2. Plug the coil cord for the Electronic Controller into the HBX port (see Figure 5).

Using Logical Mapping to Identify Steps in a Task

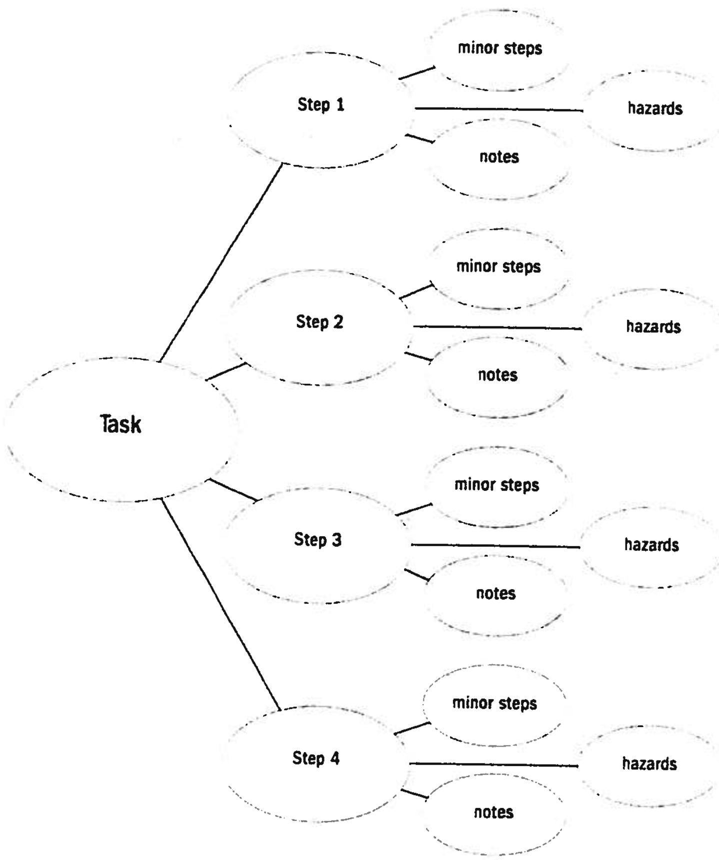


Figure 20.3: With logical mapping, the task is broken down into major and minor steps. Places where notes and hazard statements might appear are also noted.

In most steps, the verb should come first in the sentence. This puts the action up front, while keeping the pattern of the steps consistent. The "you" in these sentences is not stated, but rather implied ("*You* place the telescope in an upright position").

STATE ONE ACTION PER STEP Each step should express only one action (Figure 20.9). You might be tempted to state two smaller actions in one step, but your readers will appreciate following each step separately.

Ineffective

2. Place the telescope securely on its side as shown in Figure 4 and open the battery compartment by simultaneously depressing the two release latches.

Revised

2. Place the telescope securely on its side as shown in Figure 4.
3. Open the battery compartment by simultaneously depressing the two release latches.

However, when two actions must be completed at the same time, you should put them in the same sentence.

6. Insert a low-power eyepiece (e.g., 26mm) into the eyepiece holder and tighten the eyepiece thumbscrew.

Instructions with Sequentially Ordered Steps

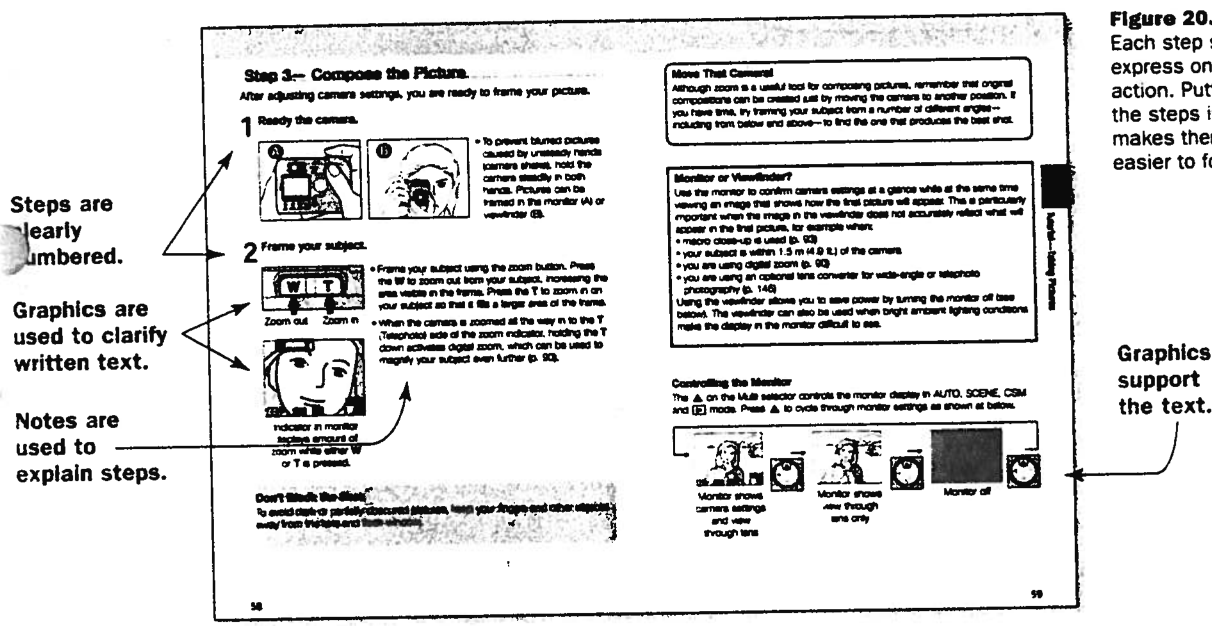


Figure 20.9: Each step shou express one action. Putting the steps in a makes them easier to follow

Source: Nikon, Cool Pix 885 *guide.*

You should state two actions in one step only when the two actions are dependent on each other. In other words, completion of one action should require the other action to be handled at the same time.

KEEP THE STEPS CONCISE: Use concise phrasing to describe each step. Short sentences are preferred so readers can remember each step while they work.

7. Adjust the focus of the telescope with the focusing knob.

8. Center the observed object in the lens.

If your sentences seem too long, consider moving some information into a follow-up "Note" or "Comment" that elaborates on the step.

NUMBER THE STEPS In most kinds of documentation, steps are presented in a numbered list. Start with the number 1 and mark each step sequentially with its own number. Notes or warnings should not be numbered, because they do not state steps to be followed.

Incorrect

9. Aim the telescope with the electronic controller.

10. Your controller is capable of moving the telescope in several different directions. It will take practice to properly aim the telescope.

There is no action in step 10 above, so a number should not be used.

Correct

9. Aim the telescope with the electronic controller.

   Your controller is capable of moving the telescope in several different directions. It will take practice to properly aim the telescope.

An important exception to this "number only steps" guideline involves the numbering of procedures and specifications. Procedures and specifications often use an itemized numbering system in which lists of cautions or notes are "nested" within lists of steps.

10.2.1 Putting on Clean Room Gloves, Hood, and Coveralls

10.2.1.1 Put on a pair of clean room gloves so they are fully extended over the arm and the coverall sleeves. Glove liners are optional.

10.2.1.2 Put a face/beard mask on, completely covering the mouth and nose.

Cautions and notes receive numbers in some procedures and specifications.

10.2.1.2.1 *Caution: No exposed hair is allowed in the fab.*

10.2.1.2.2 *Caution: Keep your nose covered at all times while in the fab.*

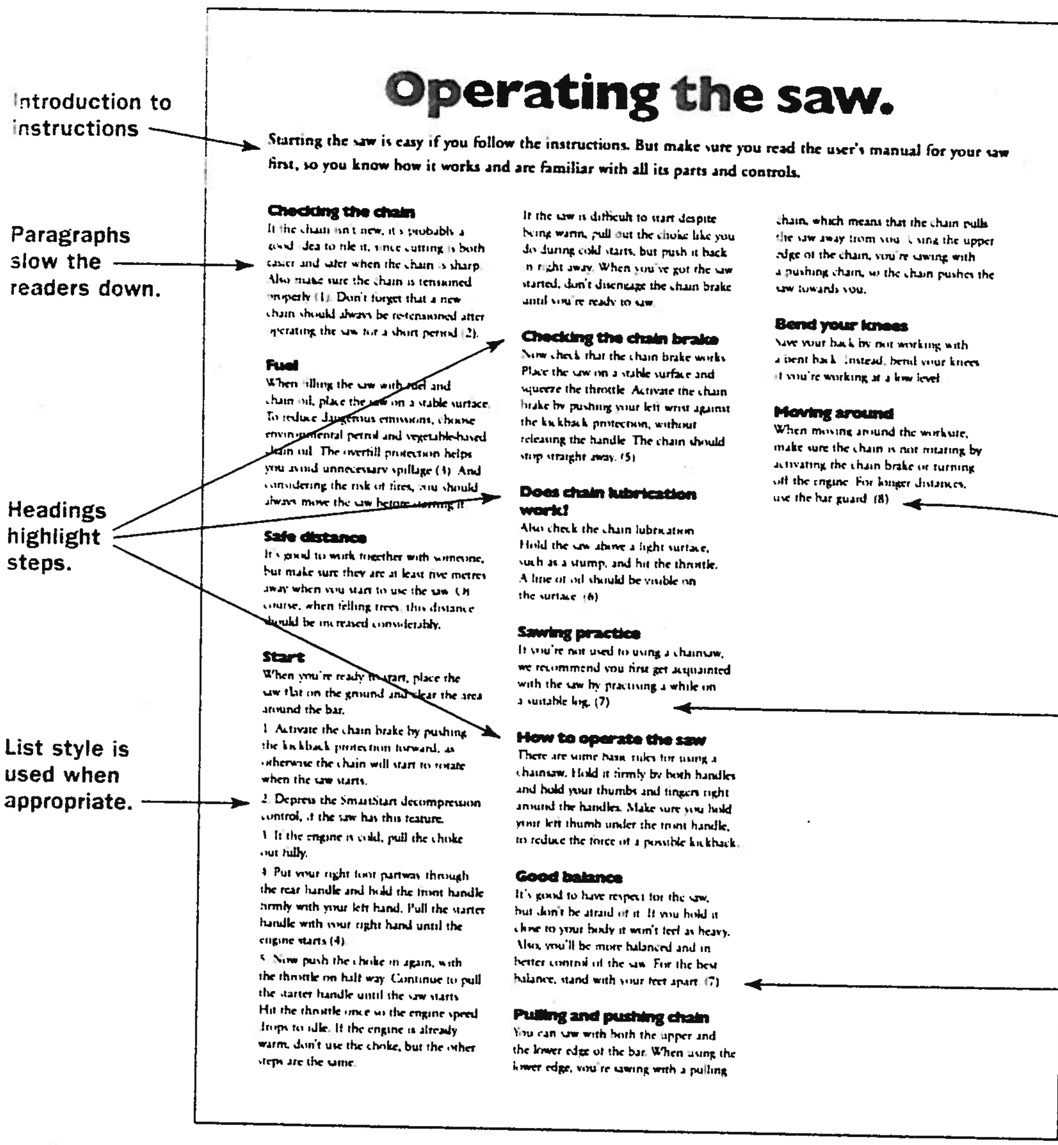
10.2.1.2.3 *Note: Do not wear the beard cover as a face mask. A beard cover should be used with a face mask to cover facial hair.*

10.2.1.3 Put on coveralls.

In specifications, comments and hazard statements receive a number. The purpose for this advanced numbering scheme is to make items in the documentation easier to reference.

In some cases, you may also want to use *paragraph style* to describe the steps (Figure 20.10). In these situations, you can use headings or sequential transitions to

Paragraph Style Instructions



Operating the saw.

Starting the saw is easy if you follow the instructions. But make sure you read the user's manual for your saw first, so you know how it works and are familiar with all its parts and controls.

Checking the chain
If the chain isn't new, it's probably a good idea to file it, since cutting is both easier and safer when the chain is sharp. Also make sure the chain is tensioned properly (1). Don't forget that a new chain should always be retensioned after operating the saw for a short period (2).

Fuel
When filling the saw with fuel and chain oil, place the saw on a stable surface. To reduce dangerous emissions, choose environmental petrol and vegetable-based chain oil. The overfill protection helps you avoid unnecessary spillage (3). And considering the risk of fires, you should always move the saw before starting it.

Safe distance
It's good to work together with someone, but make sure they are at least five metres away when you start to use the saw. Of course, when felling trees, this distance should be increased considerably.

Start
When you're ready to start, place the saw flat on the ground and clear the area around the bar.

1. Activate the chain brake by pushing the kickback protection forward, as otherwise the chain will start to rotate when the saw starts.
2. Depress the SmartStart decompression control, if the saw has this feature.
3. If the engine is cold, pull the choke out fully.
4. Put your right foot partway through the rear handle and hold the front handle firmly with your left hand. Pull the starter handle with your right hand until the engine starts (4).
5. Now push the choke in again, with the throttle on half way. Continue to pull the starter handle until the saw starts. Hit the throttle once so the engine speed drops to idle. If the engine is already warm, don't use the choke, but the other steps are the same.

If the saw is difficult to start despite being warm, pull out the choke like you do during cold starts, but push it back in right away. When you've got the saw started, don't disengage the chain brake until you're ready to saw.

Checking the chain brake
Now check that the chain brake works. Place the saw on a stable surface and squeeze the throttle. Activate the chain brake by pushing your left wrist against the kickback protection, without releasing the handle. The chain should stop straight away. (5)

Does chain lubrication work?
Also check the chain lubrication. Hold the saw above a light surface, such as a stump, and hit the throttle. A line of oil should be visible on the surface. (6)

Sawing practice
If you're not used to using a chainsaw, we recommend you first get acquainted with the saw by practising a while on a suitable log. (7)

How to operate the saw
There are some basic rules for using a chainsaw. Hold it firmly by both handles and hold your thumbs and fingers right around the handles. Make sure you hold your left thumb under the front handle, to reduce the force of a possible kickback.

Good balance
It's good to have respect for the saw, but don't be afraid of it. If you hold it close to your body it won't feel as heavy. Also, you'll be more balanced and in better control of the saw. For the best balance, stand with your feet apart. (7)

Pulling and pushing chain
You can saw with both the upper and the lower edge of the bar. When using the lower edge, you're sawing with a pulling chain, which means that the chain pulls the saw away from you. Using the upper edge of the chain, you're sawing with a pushing chain, so the chain pushes the saw towards you.

Bend your knees
Save your back by not working with a bent back. Instead, bend your knees if you're working at a low level.

Moving around
When moving around the worksite, make sure the chain is not rotating by activating the chain brake or turning off the engine. For longer distances, use the bar guard. (8)

Source: Husqvarna.

Figure 20.10: Paragraph style often takes up less space. It [is a] little harder to read at a glance, but the style [can] often be more personal. In [these] instructions [the] pictures show helpful details [to] clarify the steps.

highlight the steps. Numerical transitions ("first," "second," "third," "finally") are best in most cases. In shorter sets of steps, you might use transitions like "then," "next," "5 minutes later," and "finally" to mark the actions. In Figure 20.10, headings are used to mark transitions among the major steps.

Figure 20.10: *continued*

Readers often find paragraph style harder to follow because they cannot easily find their place again in the list of instructions. In some cases, though, paragraph style takes up less space and sounds more friendly and conversational.

> **AT A GLANCE**
>
> **Writing Effective Steps**
>
> - Use command voice.
> - State one action per step.
> - Keep the steps concise.
> - Number the steps.
> - Add comments, notes, or examples.
> - Provide feedback.
> - Refer to the graphics.

ADD COMMENTS, NOTES, OR EXAMPLES After each step, you can include additional comments or examples that will help readers complete the action. Comments after steps might include additional advice or definitions for less experienced readers. Or, comments might provide troubleshooting advice in case the step did not work out.

3. Locate a place to set your telescope.

Finding a suitable place to set up your telescope can be tricky. A paved area is optimal to keep the telescope steady. If a paved area is not available, find a level place where you can firmly set your telescope's tripod in the soil.

Comments and examples are often written in the "you" style to maintain a positive tone.

PROVIDE FEEDBACK After a difficult step or group of steps, you might offer a paragraph of feedback to help readers assess their progress.

> When you finish these steps, the barrel of your telescope should be pointed straight up. The tripod should be stable so that it does not teeter when touched. The legs of the tripod should be planted firmly on the ground.

REFER TO THE GRAPHICS In the steps, refer readers to any accompanying graphics. A simple statement like, "See Figure 4" or "(Figure 4)" will notify readers that a graphic is available that illustrates the step. After reading the step, they can look at the graphic for help in completing the step properly.

In some cases, graphics are not labeled. In these situations, the graphic should appear immediately next to the step or below it so readers know which visual goes with each step.

Revising, Editing, and Proofreading

In many ways, the documentation reflects the quality of the product, service, and manufacturer. If the documentation looks unprofessional or has errors, readers will doubt the quality of the company behind it. So, you should leave yourself plenty of time for revising, editing, and proofreading. Also, leave yourself some time to user-test your documentation on some real readers.

Revising for Content, Style, and Design

When you are finished drafting, spend time critically studying the document's content, style, and design.

- The content needs to be complete and include all the information readers need to complete the task. Look for places where steps are missing or unclear. Identify steps that would be clearer with a follow-up note or comment. ~~Identify places where hazard statements are needed.~~
- The style should be concise and clear. Keep sentences, especially commands, short and to the point. Where possible, replace complex words with simpler, plainer terms. Meanwhile, use words that reflect readers' attitudes or emotions as they will be using your documentation.
- The design should enhance readers' ability to follow the steps. Make sure you have used headings, notes, and graphics consistently. The page layout should also be consistent from the first page to the last.

Link

For more advice on revision and editing, see Chapter 12, page 306.

As you revise, ask yourself whether the content, style, and design are appropriate for the readers you defined in your reader profile. Your documentation should be usable by the least experienced, least knowledgeable person among your readers.

User-Testing with Sample Readers

User-testing your documentation on actual people can be done informally or formally, as discussed in Chapter 12 of this book.

- *Informal user-testing* can be used for in-house or less important forms of documentation. You should ask at least your supervisor or a few co-workers to look over your documentation before making it available to others. Often, your colleagues can help you identify missing content, ambiguous sentences, and ineffective graphics.
- *Formal user-testing* involves finding real users of the documentation and observing them as they try to follow the steps. Often, formal user-testing includes videotaping the subjects as they use the documentation.

Editing and Proofreading

Finally, you should carefully edit and proofread your documentation. The text should be clearly written and error free, whether the document is being sent with a product, included in a book of procedures, or placed in the specifications file.

You should challenge the content, organization, style, and design. Be your own harshest critic. Then, while proofreading, pay close attention to grammar and spelling. Flawless documentation will strengthen your readers' faith in you and your company.

CHAPTER REVIEW

- Documentation describes step-by-step how to complete a task.
- Basic features of documentation include a specific and precise title; an introduction; a list of parts, tools, and conditions required; sequentially ordered steps; graphics; safety information; a conclusion; and troubleshooting information.
- Determine the rhetorical situation by asking the Five-W and How Questions and analyzing the document's subject, purpose, readers, and context of use.
- Organize and draft your documentation step-by-step, breaking tasks down into their major and minor actions.
- Safety information should (1) identify the hazard, (2) state the level of risk (Danger, Warning, or Caution), and (3) offer suggestions for avoiding injury or damage.
- The style of your documentation does not need to be dry and boring. Try to use a style that reflects or counters readers' attitudes as they follow the steps.
- Graphics offer important support for the written text. They should be properly labeled by number and inserted on the page where they are referenced.
- While revising, user-testing your documentation with sample readers is an effective way to work out any bugs and locate places for improvement. Your observations of these sample readers should help you revise the document.

EXERCISES AND PROJECTS

Individual or Team Projects

1. Find an example of documentation in your home or workplace. Using concepts discussed in this chapter, develop a set of criteria to evaluate its content, organization, style, and design. Then, write a two-page memo to your instructor in which you analyze the documentation. Highlight any strengths and make suggestions for improvements.

2. In your home or workplace, find an ineffective set of instructions. First, identify its weaknesses in content, organization, style, and design. Then, revise the instructions to make them easier to use. Write a cover memo to your instructor in which you discuss the ways you revised and improved the set of instructions.

3. Turn the instructions for playing Klondike at the end of this chapter into a numbered list and use graphics to support the text. You can make minor changes to the wording of the text; however, try to keep the written text intact as much as possible.

4. On the Internet or in your home, find information on first aid (handling choking, treating injuries, using CPR, handling drowning, treating shock, dealing with alcohol or drug overdoses). Then, turn this information into a text that is specifically aimed at college students living on campus. You should keep in mind that these

5.3 A Mathematician's Lament

A Mathematician's Lament

by Paul Lockhart

A musician wakes from a terrible nightmare. In his dream he finds himself in a society where music education has been made mandatory. "We are helping our students become more competitive in an increasingly sound-filled world." Educators, school systems, and the state are put in charge of this vital project. Studies are commissioned, committees are formed, and decisions are made— all without the advice or participation of a single working musician or composer.

Since musicians are known to set down their ideas in the form of sheet music, these curious black dots and lines must constitute the "language of music." It is imperative that students become fluent in this language if they are to attain any degree of musical competence; indeed, it would be ludicrous to expect a child to sing a song or play an instrument without having a thorough grounding in music notation and theory. Playing and listening to music, let alone composing an original piece, are considered very advanced topics and are generally put off until college, and more often graduate school.

As for the primary and secondary schools, their mission is to train students to use this language— to jiggle symbols around according to a fixed set of rules: "Music class is where we take out our staff paper, our teacher puts some notes on the board, and we copy them or transpose them into a different key. We have to make sure to get the clefs and key signatures right, and our teacher is very picky about making sure we fill in our quarter-notes completely. One time we had a chromatic scale problem and I did it right, but the teacher gave me no credit because I had the stems pointing the wrong way."

In their wisdom, educators soon realize that even very young children can be given this kind of musical instruction. In fact it is considered quite shameful if one's third-grader hasn't completely memorized his circle of fifths. "I'll have to get my son a music tutor. He simply won't apply himself to his music homework. He says it's boring. He just sits there staring out the window, humming tunes to himself and making up silly songs."

In the higher grades the pressure is really on. After all, the students must be prepared for the standardized tests and college admissions exams. Students must take courses in Scales and Modes, Meter, Harmony, and Counterpoint. "It's a lot for them to learn, but later in college when they finally get to hear all this stuff, they'll really appreciate all the work they did in high school." Of course, not many students actually go on to concentrate in music, so only a few will ever get to hear the sounds that the black dots represent. Nevertheless, it is important that every member of society be able to recognize a modulation or a fugal passage, regardless of the fact that they will never hear one. "To tell you the truth, most students just aren't very good at music. They are bored in class, their skills are terrible, and their homework is barely legible. Most of them couldn't care less about how important music is in today's world; they just want to take the minimum number of music courses and be done with it. I guess there are just music people and non-music people. I had this one kid, though, man was she sensational! Her sheets were impeccable— every note in the right place, perfect calligraphy, sharps, flats, just beautiful. She's going to make one hell of a musician someday."

Waking up in a cold sweat, the musician realizes, gratefully, that it was all just a crazy dream. "Of course!" he reassures himself, "No society would ever reduce such a beautiful and meaningful art form to something so mindless and trivial; no culture could be so cruel to its children as to deprive them of such a natural, satisfying means of human expression. How absurd!"

Meanwhile, on the other side of town, a painter has just awakened from a similar nightmare...

I was surprised to find myself in a regular school classroom— no easels, no tubes of paint. "Oh we don't actually apply paint until high school," I was told by the students. "In seventh grade we mostly study colors and applicators." They showed me a worksheet. On one side were swatches of color with blank spaces next to them. They were told to write in the names. "I like painting," one of them remarked, "they tell me what to do and I do it. It's easy!"

After class I spoke with the teacher. "So your students don't actually do any painting?" I asked. "Well, next year they take Pre-Paint-by-Numbers. That prepares them for the main Paint-by-Numbers sequence in high school. So they'll get to use what they've learned here and apply it to real-life painting situations— dipping the brush into paint, wiping it off, stuff like that. Of course we track our students by ability. The really excellent painters— the ones who know their colors and brushes backwards and forwards— they get to the actual painting a little sooner, and some of them even take the Advanced Placement classes for college credit. But mostly we're just trying to give these kids a good foundation in what painting is all about, so when they get out there in the real world and paint their kitchen they don't make a total mess of it."

"Um, these high school classes you mentioned..."

"You mean Paint-by-Numbers? We're seeing much higher enrollments lately. I think it's mostly coming from parents wanting to make sure their kid gets into a good college. Nothing looks better than Advanced Paint-by-Numbers on a high school transcript."

"Why do colleges care if you can fill in numbered regions with the corresponding color?"

"Oh, well, you know, it shows clear-headed logical thinking. And of course if a student is planning to major in one of the visual sciences, like fashion or interior decorating, then it's really a good idea to get your painting requirements out of the way in high school."

"I see. And when do students get to paint freely, on a blank canvas?"

"You sound like one of my professors! They were always going on about expressing yourself and your feelings and things like that—really way-out-there abstract stuff. I've got a degree in Painting myself, but I've never really worked much with blank canvasses. I just use the Paint-by-Numbers kits supplied by the school board."

***

Sadly, our present system of mathematics education is precisely this kind of nightmare. In fact, if I had to design a mechanism for the express purpose of *destroying* a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done— I simply wouldn't have the imagination to come up with the kind of senseless, soul-crushing ideas that constitute contemporary mathematics education.

Everyone knows that something is wrong. The politicians say, "we need higher standards." The schools say, "we need more money and equipment." Educators say one thing, and teachers say another. They are all wrong. The only people who understand what is going on are the ones most often blamed and least often heard: the students. They say, "math class is stupid and boring," and they are right.

Mathematics and Culture

The first thing to understand is that mathematics is an art. The difference between math and the other arts, such as music and painting, is that our culture does not recognize it as such. Everyone understands that poets, painters, and musicians create works of art, and are expressing themselves in word, image, and sound. In fact, our society is rather generous when it comes to creative expression; architects, chefs, and even television directors are considered to be working artists. So why not mathematicians?

Part of the problem is that nobody has the faintest idea what it is that mathematicians do. The common perception seems to be that mathematicians are somehow connected with science— perhaps they help the scientists with their formulas, or feed big numbers into computers for some reason or other. There is no question that if the world had to be divided into the "poetic dreamers" and the "rational thinkers" most people would place mathematicians in the latter category.

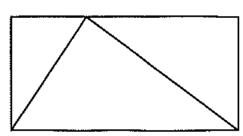
Nevertheless, the fact is that there is nothing as dreamy and poetic, nothing as radical, subversive, and psychedelic, as mathematics. It is every bit as mind blowing as cosmology or physics (mathematicians *conceived* of black holes long before astronomers actually found any), and allows more freedom of expression than poetry, art, or music (which depend heavily on properties of the physical universe). Mathematics is the purest of the arts, as well as the most misunderstood.

So let me try to explain what mathematics is, and what mathematicians do. I can hardly do better than to begin with G.H. Hardy's excellent description:

> A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*.

So mathematicians sit around making patterns of ideas. What sort of patterns? What sort of ideas? Ideas about the rhinoceros? No, those we leave to the biologists. Ideas about language and culture? No, not usually. These things are all far too complicated for most mathematicians' taste. If there is anything like a unifying aesthetic principle in mathematics, it is this: *simple is beautiful.* Mathematicians enjoy thinking about the simplest possible things, and the simplest possible things are *imaginary.*

For example, if I'm in the mood to think about shapes— and I often am— I might imagine a triangle inside a rectangular box:

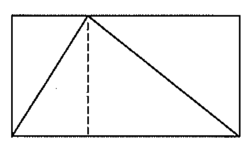


I wonder how much of the box the triangle takes up? Two-thirds maybe? The important thing to understand is that I'm not talking about this *drawing* of a triangle in a box. Nor am I talking about some metal triangle forming part of a girder system for a bridge. There's no ulterior practical purpose here. I'm just *playing*. That's what math is— wondering, playing, amusing yourself with your imagination. For one thing, the question of how much of the box the triangle takes up doesn't even make any *sense* for real, physical objects. Even the most carefully made physical triangle is still a hopelessly complicated collection of jiggling atoms; it changes its size from one minute to the next. That is, unless you want to talk about some sort of *approximate* measurements. Well, that's where the aesthetic comes in. That's just not simple, and consequently it is an ugly question which depends on all sorts of real-world details. Let's leave that to the scientists. The *mathematical* question is about an imaginary triangle inside an imaginary box. The edges are perfect because I want them to be— that is the sort of object I prefer to think about. This is a major theme in mathematics: things are what you want them to be. You have endless choices; there is no reality to get in your way.

On the other hand, once you have made your choices (for example I might choose to make my triangle symmetrical, or not) then your new creations do what they do, whether you like it or not. This is the amazing thing about making imaginary patterns: they talk back! The triangle takes up a certain amount of its box, and I don't have any control over what that amount is. There is a number out there, maybe it's two-thirds, maybe it isn't, but I don't get to say what it is. I have to *find out* what it is.

So we get to play and imagine whatever we want and make patterns and ask questions about them. But how do we answer these questions? It's not at all like science. There's no experiment I can do with test tubes and equipment and whatnot that will tell me the truth about a figment of my imagination. The only way to get at the truth about our imaginations is to use our imaginations, and that is hard work.

In the case of the triangle in its box, I do see something simple and pretty:



If I chop the rectangle into two pieces like this, I can see that each piece is cut diagonally in half by the sides of the triangle. So there is just as much space inside the triangle as outside. That means that the triangle must take up exactly half the box!

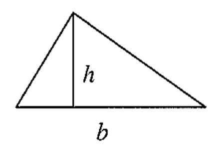
This is what a piece of mathematics looks and feels like. That little narrative is an example of the mathematician's art: asking simple and elegant questions about our imaginary creations, and crafting satisfying and beautiful explanations. There is really nothing else quite like this realm of pure idea; it's fascinating, it's fun, and it's free!

Now where did this idea of mine come from? How did I know to draw that line? How does a painter know where to put his brush? Inspiration, experience, trial and error, dumb luck. That's the art of it, creating these beautiful little poems of thought, these sonnets of pure reason. There is something so wonderfully transformational about this art form. The relationship between the triangle and the rectangle was a mystery, and then that one little line made it obvious. I couldn't see, and then all of a sudden I could. Somehow, I was able to create a profound simple beauty out of nothing, and change myself in the process. Isn't that what art is all about?

This is why it is so heartbreaking to see what is being done to mathematics in school. This rich and fascinating adventure of the imagination has been reduced to a sterile set of "facts" to be memorized and procedures to be followed. In place of a simple and natural question about shapes, and a creative and rewarding process of invention and discovery, students are treated to this:

Triangle Area Formula:

$$A = 1/2\, b\, h$$



"The area of a triangle is equal to one-half its base times its height." Students are asked to memorize this formula and then "apply" it over and over in the "exercises." Gone is the thrill, the joy, even the pain and frustration of the creative act. There is not even a *problem* anymore. The question has been asked and answered at the same time— there is nothing left for the student to do.

Now let me be clear about what I'm objecting to. It's not about formulas, or memorizing interesting facts. That's fine in context, and has its place just as learning a vocabulary does— it helps you to create richer, more nuanced works of art. But it's not the *fact* that triangles take up half their box that matters. What matters is the beautiful *idea* of chopping it with the line, and how that might inspire other beautiful ideas and lead to creative breakthroughs in other problems— something a mere statement of fact can never give you.

By removing the creative process and leaving only the results of that process, you virtually guarantee that no one will have any real engagement with the subject. It is like *saying* that Michelangelo created a beautiful sculpture, without letting me *see* it. How am I supposed to be inspired by that? (And of course it's actually much worse than this— at least it's understood that there *is* an art of sculpture that I am being prevented from appreciating).

By concentrating on *what*, and leaving out *why*, mathematics is reduced to an empty shell. The art is not in the "truth" but in the explanation, the argument. It is the argument itself which gives the truth its context, and determines what is really being said and meant. Mathematics is *the art of explanation*. If you deny students the opportunity to engage in this activity— to pose their own problems, make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs— you deny them mathematics itself. So no, I'm not complaining about the presence of facts and formulas in our mathematics classes, I'm complaining about the lack of *mathematics* in our mathematics classes.

If your art teacher were to tell you that painting is all about filling in numbered regions, you would know that something was wrong. The culture informs you— there are museums and galleries, as well as the art in your own home. Painting is well understood by society as a medium of human expression. Likewise, if your science teacher tried to convince you that astronomy is about predicting a person's future based on their date of birth, you would know she was crazy— science has seeped into the culture to such an extent that almost everyone knows about atoms and galaxies and laws of nature. But if your math teacher gives you the impression, either expressly or by default, that mathematics is about formulas and definitions and memorizing algorithms, who will set you straight?

The cultural problem is a self-perpetuating monster: students learn about math from their teachers, and teachers learn about it from their teachers, so this lack of understanding and appreciation for mathematics in our culture replicates itself indefinitely. Worse, the perpetuation of this "pseudo-mathematics," this emphasis on the accurate yet mindless manipulation of symbols, creates its own culture and its own set of values. Those who have become adept at it derive a great deal of self-esteem from their success. The last thing they want to hear is that math is really about raw creativity and aesthetic sensitivity. Many a graduate student has come to grief when they discover, after a decade of being told they were "good at math," that in fact they have no real mathematical talent and are just very good at following directions. Math is not about following directions, it's about making new directions.

And I haven't even mentioned the lack of mathematical criticism in school. At no time are students let in on the secret that mathematics, like any literature, is created by human beings for their own amusement; that works of mathematics are subject to critical appraisal; that one can have and develop mathematical *taste*. A piece of mathematics is like a poem, and we can ask if it satisfies our aesthetic criteria: Is this argument sound? Does it make sense? Is it simple and elegant? Does it get me closer to the heart of the matter? Of course there's no criticism going on in school— there's no art being done to criticize!

Why don't we want our children to learn to do mathematics? Is it that we don't trust them, that we think it's too hard? We seem to feel that they are capable of making arguments and coming to their own conclusions about Napoleon, why not about triangles? I think it's simply that we as a culture don't know what mathematics is. The impression we are given is of something very cold and highly technical, that no one could possibly understand— a self-fulfilling prophesy if there ever was one.

It would be bad enough if the culture were merely ignorant of mathematics, but what is far worse is that people actually think they *do* know what math is about— and are apparently under the gross misconception that mathematics is somehow useful to society! This is already a huge difference between mathematics and the other arts. Mathematics is viewed by the culture as some sort of tool for science and technology. Everyone knows that poetry and music are for pure enjoyment and for uplifting and ennobling the human spirit (hence their virtual elimination from the public school curriculum) but no, math is *important*.

SIMPLICIO: Are you really trying to claim that mathematics offers no useful or practical applications to society?

SALVIATI: Of course not. I'm merely suggesting that just because something happens to have practical consequences, doesn't mean that's what it is *about*. Music can lead armies into battle, but that's not why people write symphonies. Michelangelo decorated a ceiling, but I'm sure he had loftier things on his mind.

6. Practice Problems

6.1 Spherical Geometry Problems

Spherical Geometry Practice Probems

TRUE/FALSE

1. The shortest point between two points on a sphere is on the arc of a great circle.

2. If two great circles intersect to form a right angle, they are perpendicular.

3. On a sphere there is only one distance that can be measured between two points.

4. A great circle is infinite.

5. Every two unique geodesics on a sphere must cross at two points

6. Two perpendicular ‘lines’ on a sphere create four right angles.

7. A triangle on a sphere may have three right angles.

8. On a sphere if two ’lines’ are perpendicular to a given line, the two lines are parallel.

9. Rectangles can be drawn on a sphere.

1. true
2. true
3. false (the great circle connects the points in two ways)
4. false
5. true
6. false (there is another set of 4 at the other point the two great circles intersect)
7. true
8. false
9. false