Midterm TMath 403 Spring 2026

1 Directions:

Use any resources available to prepare, including your classmates, the TLC, internet, or me. You should plan, discuss, and debate answers with anyone that is willing to engage. **You are required to cite your sources and collaborators.**

- Sign up for a 25 minute interview slot at: `https://calendly.com/r-e-vanderpool/403-midterm`
- The assessment is closed book but you make use a one-sided 8.5" by 11" sheet of notes with definitions and theorems. No prepared solutions.
- Location: SNO 236B
- Please arrive 5 minutes early and get your materials prepared so that we can start promptly. You will have a maximum of 25 minutes for the midterm.

1.1 Grading Rubric

The exam consists of three questions. The following rubric will be used for each question.

95%	Well-executed. Thorough discussion. All points are well supported. Two or fewer minor errors. No nontrivial errors.
85%	Generally well-executed. Several minor errors; or a nontrivial mathematical error that gets corrected when identified.
75%	Uncorrected nontrivial error; or several nontrivial errors that get corrected when identified; or error in fundamental understanding that gets corrected when identified.
60%	Error in understanding of fundamental concept that does not get corrected.
0	No evidence of preparation or understanding.

- (20 pts) The first question will be **your choice**.
- (20 pts) I choose the second question from the ones remaining.
  - You may pass on my choice once for a 5 percentage point penalty on the overall Midterm score. If the pass is used, I select another problem.
- (10 pts) Last problem that has an element of chance in it.

2 Midterm Questions:

1. Determine if the following algebraic objects satisfy algebraic conditions enumerated below. Be prepared to explain your choices with examples (SLO #4). If none of the objects satisfy the given algebraic condition, be prepared to provide a different example, if possible.

   $\mathbb{Z}_{14} \times \mathbb{Z}_7$ $\quad$ $\mathbb{Q}[[x]]$ $\quad$ $\mathbb{Z}_3 \times \mathbb{Z}_3$ $\quad$ $\mathbb{Z}_{29}$

   (a) Does not have a commutative ring structure.

   (b) Has a subgroup that is not an ideal.

   (c) Is an Integral Domain.

   (d) Is an infinite ring with at least 3 zero divisors.

   (e) Is not a field.

2. Prove or provide a counterexample for the following statement: "The ideal $I = (x, 5)$ is maximal in $\mathbb{Z}[x]$."

3. Let $f(x) \in \mathbb{F}_3[x]$. We will examine $\mathbb{F}_3[x]/(f(x))$ performing addition & multiplication between elements as well as determining what kind of algebraic object it is (field? ID?).

   To define $f(x)$, use the first three letters of your first name to build $f(x)$ of the form $a_0+a_1x+a_2x^2$. Specifically, use the table below to let a_0 be the number that corresponds to the first letter in your first name. For Ruth then "R" would set $a_0 = 0$. Let a_1 be the number that corresponds to the second letter and a_2 correspond to the third letter. For Ruth then $a_1 = 1$ and $a_2 = 2$, thus the polynomial for Ruth is $f(x) = 0 + 1x + 2x^2$.

1	A	D	G	J	M	P	S	V	Y	L	U	
2	B	E	H	K	N	Q	T	W	Z	C	I	O
0		F				R		X				

4. Let $R = \mathbb{Z}[\sqrt{3}]$ and $\mathrm{Frac}(R)$ be the field of fractions of R. Describe the elements in $\mathrm{Frac}(R)$ and be prepared to add, subtract, multiply, and divide different elements from $\mathrm{Frac}(R)$.

Everyone will do the last question which examines maps from $\mathbb{Z}_{10\cdot a}$ to $\mathbb{Z}_{10\cdot b}$. You will role a die to determine a and I will roll the die to determine b. The questions will be largely impromptu investigating SLO #2, but below are some of the questions to give you an idea of what might be asked:

1. Build a map and verify it is a well-defined ring homomorphism.

2. Can we build a one-to-one ring homomorphism between the rings that is not onto?

3. Can we build an onto ring homomorphism between the rings that is not one-to-one?

4. For a ring homomorphism ϕ, what is the factor ring $(\mathbb{Z}_{10\cdot a}/\ker(\phi))$ isomorphic to?