Final TMath 403 Spring 2026

1 Directions:

Use any resources available to prepare, including your classmates, the TLC, internet, or me. You should plan, discuss, and debate answers with anyone that is willing to engage. **You are required to cite your sources and collaborators.**

- Sign up for a 25 minute interview slot at: `https://calendly.com/r-e-vanderpool/403-midterm`
- The assessment is closed book but you make use a two-sided 8.5" by 11" sheet of notes with definitions and theorems. No prepared solutions.
- Location:
  - Thursday June 4th: SNO 222C
  - Friday June 5th: MDS 303C
  - Tuesday June 9th: SNO 236B
- Please arrive 5 minutes early and get your materials prepared so that we can start promptly. You will have a maximum of 25 minutes for the final.

1.1 Grading Rubric

The exam consists of three questions. The following rubric will be used for each question.

95%	Well-executed. Thorough discussion. All points are well supported. Two or fewer minor errors. No nontrivial errors.
85%	Generally well-executed. Several minor errors; or a nontrivial mathematical error that gets corrected when identified.
75%	Uncorrected nontrivial error; or several nontrivial errors that get corrected when identified; or error in fundamental understanding that gets corrected when identified.
60%	Error in understanding of fundamental concept that does not get corrected.
0	No evidence of preparation or understanding.

- (20 pts) The first question will be **your choice**.
- (20 pts) I choose the second question from the ones remaining.
  - You may pass on my choice once for a 5 percentage point penalty on the overall Final exam score. If the pass is used, I select another problem.
- (10 pts) Last problem that has an element of chance in it.

2 Final Questions:

1. Determine if the following algebraic objects satisfy algebraic conditions enumerated below. Be prepared to explain your choices with examples (SLO #4). If none of the objects satisfy the given algebraic condition, be prepared to provide a different example, if possible.

   $\mathbb{Z}_{14}$ $\qquad$ $\mathbb{Q}(\sqrt[3]{11})$ $\qquad$ $\mathbb{Z}_5 \times \mathbb{Z}_{15}$ $\qquad$ $\mathbb{Z}_{29}[x]$

   (a) Is a commutative ring but not a principal ideal domain.
   (b) Is a ring without unity.
   (c) Is a Euclidean domain but not a field.
   (d) Is an finite ring with at least 4 zero divisors.
   (e) Is a field.

2. Let n be the number of letters in your first name and m be the number of letters in your last name. For example, "Ruth" would correspond to $n = 4$.

   (a) Describe $\mathbb{Q}(\sqrt{n} + \sqrt{m})$ (SLO #3).
   (b) Find $[\mathbb{Q}(\sqrt{n} + \sqrt{m}) : \mathbb{Q}]$.

3. Define the set of constructible numbers, C, any binary operators, and identify what kind of algebraic object it is. Be prepared to provide examples of numbers that are in C and examples of numbers that are not.

4. Let $F = \mathbb{Q}(\sqrt{2}, \sqrt{5})$. We write an element of F with the basis $\{1, \sqrt{2}, \sqrt{5}, \sqrt{10}\}$. For $a, b, c, d \in \mathbb{Q}$, define:

   $\tau : F \to F$ by $\tau(a + b\sqrt{2} + c\sqrt{5} + d\sqrt{10}) = a - b\sqrt{2} + c\sqrt{5} - d\sqrt{10}$

   $\sigma : F \to F$ by $\sigma(a + b\sqrt{2} + c\sqrt{5} + d\sqrt{10}) = a + b\sqrt{2} - c\sqrt{5} - d\sqrt{10}$

   (a) Verify τ is a field isomorphism.
   (b) Prove σ and τ generate a group inside the set of isomorphisms from F to F. Provide a Cayley table or Cayley Diagram.

Everyone will do the last question which examines maps from $\mathbb{Z}_{6\cdot a}$ to $\mathbb{Z}_{6\cdot b}$. You will role a die to determine a and I will roll the die to determine b. The questions will be largely impromptu investigating SLO #2, but below are some of the questions to give you an idea of what might be asked:

- Build a map and verify it is a well-defined ring homomorphism.
- Can we build a one-to-one ring homomorphism between the rings that is not onto?
- Can we build an onto ring homomorphism between the rings that is not one-to-one?
- For a ring homomorphism ϕ, what is the factor ring $(\mathbb{Z}_{6\cdot a}/\ker(\phi))$ isomorphic to?