TMath 403

This section is to be taken home, completed, and turned in by 8:00pm Wednesday Apr 17th. There is no time limit and you do not need to type up your solutions to get full marks although the answers should be well edited and readable.

You may discuss this problem with anyone else from the class and use the class resources posted on Canvas. You may not consult anyone or any resource that is not affiliated with the class such as tutors, websites, or other textbooks.

1. [3] Let n be the number of letters in your first name. Let m be the number of letters in your last name. Find all $\phi : \mathbb{Z}_m \to \mathbb{Z}_n$ group homomorphisms and all ring homomorphisms. Justify that you have found them all.

2. Consider $\mathbb{Z}_5[x]$

   (a) [2] Identify what kind of algebraic object $\mathbb{Z}_5[x]\big/\langle x^2 + 1\rangle$ is. (Group? Abelian Group? Ring? Commutative Ring? with Multiplicative Identity/unity? Integral Domain? Field?) Briefly justify your answer.

   (b) [3] Identify what kind of algebraic object $\mathbb{Z}_5[x]\big/(x^2 + 1)$ is. (Group? Abelian Group? Ring? Commutative Ring? with Multiplicative Identity/unity? Integral Domain? Field?) Briefly justify your answer.

   (c) [2] Find a representative of $(3x + 4) \cdot (x + 1)$ in $\mathbb{Z}_5[x]\big/(x^2 + 1)$ that has degree less than 2, if possible.