Constructions (with straightedges & compass)

We investigate an *old* question. So old, it was posed before the base ten number system was perfected... and reliable rulers. For these problems assume your only tools are a straightedge (with no marks on it) and a compass that can be used in the following three ways:

- Given two points, a straight line can be drawn joining them.
- A straight line segment can be extended indefinitely.
- Given a straight line segment, a circle can be drawn with the segment as radius and one endpoint as the center.

1. Let the length of the line segment below have measure 1. Create a line of length $1 + 1$.

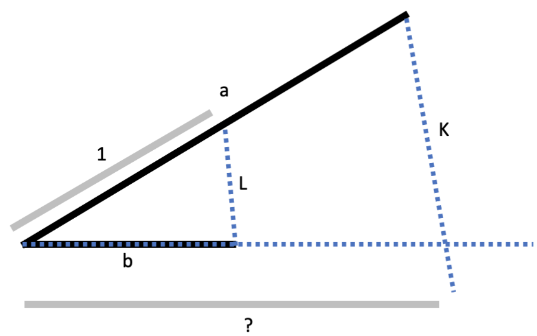
2. Consider the two line segments below. The top has length a, and the bottom has length b. Create a line of length $a + b$. Briefly describe the procedure.

3. Could you create a line of length $a - b$?

Definition 1. *A number* α *is* constructible *if we can construct a line segment of length* $|\alpha|$ *in a finite number of steps from a segment of unit length by using a straightedge and compass.*

4. Given the above exercises, what algebraic structure can we easily verify for the set of constructible numbers (group? ring? field? vector space?)

5. Let's start working on another binary operator. For now let us assume $a > 1$ and consider the triangles below. The construction was such that L and K are parallel. Note that the smaller triangle has two edges of lengths 1 and b and one of the sides of the larger triangle is a. Find the length of the bigger triangle marked with "?"



6. Given length a, could you construct $\frac{1}{a}$? If so, outline how.

7. Given the above exercises, what algebraic structure can we verify for the set of constructible numbers (group? ring? field? vector field?)