402 Midterm Practice 2026

Note: This is a practice midterm and is intended only for study purposes in class. The actual exam will contain different questions, will be longer, and may have a different layout.

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. The set of all pairs $(x, y) \in \mathbb{R}^2$ such that $y \neq 0$, under the operation $(x, y) \star (z, w) = (x + z, yw)$ forms a group.

2. If G is a group in which $(ab)^2 = a^2b^2$ for all $a \in G$ and $b \in G$, then G is abelian.

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.
(If you use a calculator, be sure to tell me.)

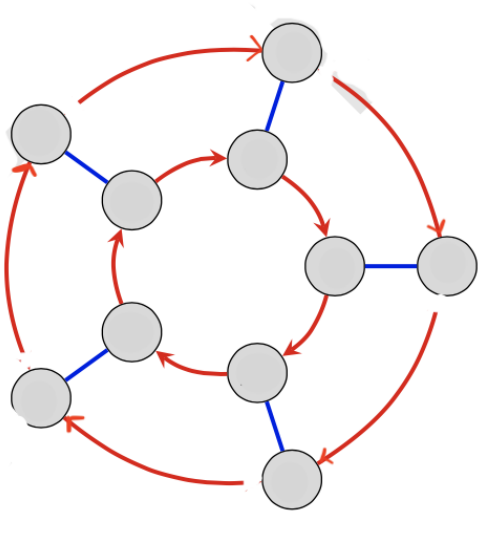
3. Let $H = \langle(1234)\rangle \leq S_4$ where S_4 is the symmetric group on four elements. Define $N_{S_4}(H) = \{\sigma \in S_4 | \sigma H \sigma^{-1} = H\}$ or $\{\sigma \in S_4 | \sigma h \sigma^{-1} \in H, \forall h \in H\}$.

   (a) Is $(12) \in N_{S_4}(H)$? Justify your answer.

   (b) Is $(12)(34) \in N_{S_4}(H)$? Justify your answer.

4. Provide a counterexample to or provide a proof for the following statement: Given a subgroup J in a group G, the set $N_G(J) = \{g \in G | gJg^{-1} = J\}$ is a subgroup in G. (Consider looking at the previous problem to help.)

5. A group G has the Cayley graph shown on the right. Let r depict the red action and b depict the blue. Answer the following questions about G:



(a) Rewrite $r^3br^3b^2r^3r^2b$ into the form r^ib^j where $i \in \{0, 1, 2, 3, 4\}$ and $j \in \{0, 1\}$.

(b) Find the orbit of r.

(c) Find a subgroup, H, of G such that the order of H is 2.

(d) Is G cyclic? Justify your answer.