2 Dimensions Postulates

definitions & theorems from Origametry by Daniel Heath.

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Postulate 1. *Any two points determine a unique line.*

Postulate 2. *Given any two distinct points on a line, there is a one-to-one correspondence, called a* ruler *between all points on the line with the real numbers that sends one of the two given points to 0, and the other to some number greater than 0. The number p assigned to a point P by the ruler is called the* coordinate*.*

Postulate 3. *Every line l determines a decompositions of the plane into 3 distinct sets: H_0, H_1, and l where:*

- *Every pair of points in one of the H_i are on the same side of l, and*
- *Every pair of points where one is in H_0 & the other is in $H_!$ are on opposite sides of l.*

Postulate 4. *Let $\overrightarrow{OA}$ be given. Then there is a 1-1 correspondence, called a* protractor*, between all rays $\overrightarrow{OB}$ with the real numbers in the interval $(-\pi, \pi]$ that sends $\overrightarrow{OA}$ to 0 and $\overrightarrow{OB}$ to some number $x \in (-\pi, \pi]$. The number p assigned to a ray $\overrightarrow{OP}$ by the protractor is called its* coordinate*. Furthermore:*

- *$\overrightarrow{OB}$ has coordinate 0 if and only if $\angle AOB$ is degenerate.*
- *$\overrightarrow{OB}$ has coordinate π if and only if $A - O - B$, that is if $\angle AOB$ is straight.*

Postulate 5. *Let $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ be distinct rays. Then $\mu(\angle AOB) + \mu(\angle BOC) = \mu(\angle AOC)$ if and only if B is in the interior of $\angle AOC$.*

Postulate 6. *Given a line l, there is a unique bijective function ϕ from the plane to the plane called a* fold with crease *l or a* reflection with mirror *l, such that:*

1. *The function ϕ leaves l unchanged*
2. *Let H_0 and H_1 be the half planes determined by l.*
   Then $A \in H_i$, implies $\phi(A) \in H_{1-i}$ for $i = 0, 1$.
3. *If $\overline{AB} \subset \overline{H_i}$, then $\phi(A)\phi(B) = AB$*
4. *If $\angle ABC \subset \overline{H_1}$, then $\mu(\angle\phi(A)\phi(B)\phi(C) = \mu(\angle ABC)$*

Postulate 7. *If two parallel lines are cut by a transversal, then alternate interior angles are congruent.*

Theorem Playfaire's Theorem. *Given a line l, and a point $A \notin l$, there is a unique line m through A and parallel to l.*