## 1 Dimensional Folding

definitions \& theorems from Origametry by Daniel Heath.
While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Consider the one dimensional foldability game: Begin with a set of two points $S_{1}=\left\{A_{0}, A_{1}\right\}$. We use the convention that the distance between the first two points is one unit and $A_{0}$ has coordinate 0 . We can create new points to append to the set $S_{n}$ by using either of the following methods:

1. We can use a point $A_{i}$ as a crease of a fold $\phi$ and apply $\phi$ to another point $A_{j}$ to create a new point $A_{n+1}=\phi\left(A_{j}\right)$.
2. We can fold a point $A_{i}$ onto the top of another such as $A_{j}$. This fold $\phi$ will create a new point $A_{n+1}$ at the crease of the fold.

Both of these methods will give us another point that we can add to our set $S_{n+1}=$ $\left\{A_{0}, A_{1}, \ldots . A_{n}, A_{n+1}\right\}$. The set $S_{\infty}$ is considered to be the set of foldable points. Use these rules for the following:

1. Find the point with coordinate $\frac{5}{8}$.
2. Find the smallest set of coordinates needed to produce the point with coordinate $\frac{5}{8}$.
3. Mark four points $A, B, C$, and $D$. Find a point $E$ so that $C-D-E$ and $D E=A B$. That is, add the length of $\overline{A B}$ to $\overline{C D}$. What is the smallest number of folds it takes you?

Postulate C-1. Let $\lambda>0$ be given. Given any two distinct points, there is a 1-1 correspondence, called a ruler, between all the points on the circle and the real numbers modulo $2 \lambda$ that sends one of the two given points to 0 an the other to some number $x>0$. The number $p$ assigned to a point $P$ is called its coordinate.

Definition 3.1. If $A$ and $B$ are arbitrary points with coordinates a and $B$ respectively, then the distance $A B$ from $A$ to $B$ is defined as:

$$
A B= \begin{cases}|a-b| & \text { if }|a-b| \leq \lambda \\ 2 \lambda-|a-b| & \text { if }|a-b|>\lambda\end{cases}
$$

4. Let $\lambda=\pi$. Find the distance between $\frac{2 \pi}{3}$ and $-\frac{2 \pi}{3}$. How about between 2 and 10 ?
5. Recall on the line that $B$ is between $A$ and $C$ if $A B+B C=A C$. Let us define between similarly on the circle. Identify three points on the circle such that no point is between the others.
