1 Dimensional Geometry

definitions & theorems from Origametry by Daniel Heath.

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Postulate L-1. *Given any two distinct points, there is a 1-1 correspondence, called a* ruler*, between all points with the real numbers that sends one of the two given to* 0 *and the other to some number* $x > 0$*. The number* p *assigned to a point* P *by the ruler is called its* coordinate*.*

Definition 2.1. *If* A *and* B *are arbitrary points with coordinates* a *and* b*, then the* distance AB*, from* A *and* B *is defined to be* $AB = |a - b|$*.*

Definition 2.2. *Let* A*,* B*, and* C *be distince points with coordinates* a*,* b*, and* c *respecitively. Then* B *is* between A *and* C *(denoted* $A{-}B{-}C$*) provided that either* $a < b < c$ *or* $c < b < a$*.*

Theorem 2.1. *If* B *is between* A *and* C*, then* $AB + BC = AC$*.*

Definition 2.3. *Given two distince points A and B we define the* segment *(denoted $\overline{AB}$) to be the set of points C so that either $C = A$, $C = B$, or $A - C - B$, i.e.*

$$\overline{AB} = \{C : C = A, C = B, \text{ or } A - C - B\}.$$

The length *of a segment is the distance between its endpoints.*

Definition 2.4. *Given two points A and B we define the* ray *(denoted $\overrightarrow{AB}$) to be the set of points C so that either $C = A$, $C = B$, $A - C - B$, or $A - B - C$, i.e.*

$$\overrightarrow{AB} = \{C : C = A, C = B, A - C - B, \text{ or } A - B - C\}.$$

Practice 2.4. *Suppose we define "points" as non-vertical lines through the origin in $\mathbb{R}^2$.*

1. Verify that Postulate L-1 is satisfied.

2. Sketch the segment between 0 and 1.

Postulate L-2. *Let O be a point. There is a bijective function ϕ from the line to the line called a* fold with crease O *such that:*

1. $\phi(O) = O$.
2. *Let $\overrightarrow{OA_0}$ and $\overrightarrow{OA_1}$ be the two rays determined by O. Then $B \in \overrightarrow{OA_i}$ implies $\phi(B) \in \overrightarrow{OA_{1-i}}$.*
3. *If segment $\overline{BC} \subset \overrightarrow{OA_i}$, then $BC = \phi(B)\phi(C)$.*

Definition 2.6. *Let A and B be distinct points, and let a and b be their respective coordinates. Let M be the point with coordinate $\frac{a+b}{2}$. Then M is called the* midpoint *of the segment $\overline{AB}$.*