Key

Exam 1 | **TMath 344** | Winter 2025

Show all your work. Reasonable supporting work must be shown to earn credit.

1. [10] Let A, B, C, and D be points with coordinates a, b, c, and d respectively. We use the notation from the text. Determine if the following make sense/*could* be true and be sure to justify your answer:

   ← A and B

   Stat (1.5) (a) $a = b$ Could be true if $A = B$ (+.5) (+1) eg

   Stat (1.5) (b) $A < b$ Does not make sense. (.5)

   (+1) [A is a point and not directly comparable whereas b is a coordinate and so does have the ability to be "larger than"

   Stat (1.5) (c) $a - b - c$ (+.5) Does not make sense.

   (+1) [The between notation is for points not coordinates

   Stat (1.5) (d) $\overline{AB} \in \overline{CD}$ (+.5) [Could if $A = B$. Then $\overline{AB} = \{A\}$ and A could be in $\overline{CD}$

   (+1) [Generally though if $A \neq B$ then $\overline{AB}$ is a subset of $\overline{CD}$ written $\overline{AB} \subseteq \overline{CD}$

   Stat (1.5) (e) $A \in CD$ (+.5) Does not make sense.

   (+1) [CD gives the length of the line segment which does not have points.

2. Consider the statement: $\exists x \in \mathbb{Z}, x^2 = 2$.

   (a) [2] (2019Exam #2) Use as few symbols as possible to interpret the meaning with words. (+.5)

   Sense (1.5) There exists an integer so that if the integer is squared we get two (+1)

   (b) [3] (HW1 #1.13) Negate the above statement symbolically without using the $\exists$ symbol.

   (+.5) $\neg(\exists x \in \mathbb{Z},\ x^2 = 2)$

   using de Morgan's Law

   sense (1.5) $\underbrace{\forall x \in \mathbb{Z}}_{(+1)},\ \underbrace{x^2 \neq 2}_{(+1)}$

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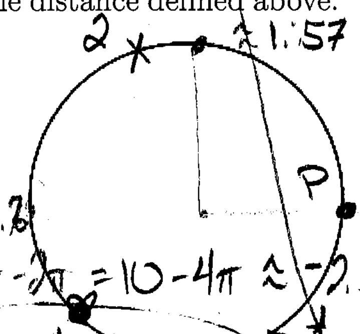
Definition 1. *If A and B are arbitrary points with coordinates a and b respectively where each is in the interval $(-\lambda, \lambda]$, then the* distance AB *from A to B is defined as:*

$$AB = \begin{cases} |a-b| & \text{if } |a-b| \leq \lambda \\ 2\lambda - |a-b| & \text{if } |a-b| > \lambda \end{cases}$$

3. Consider the circle with circumference 2π so we let $\lambda = \pi$ in the distance defined above.



(a) [3] (1dFolding Activity #4) Find the distance between the points with coordinates 2 and 10.

note (+.5) $2 \in (-\pi, \pi]$ so good coord.

use Def 1 (+.5)

(+.5) $10 \notin (-\pi, \pi] \Rightarrow 10 \equiv 10 - 2\pi \approx 3.7$

(+.5) $10 - 2\pi \notin (-\pi, \pi] \Rightarrow 10 \equiv 10 - 2\pi - 2\pi = 10 - 4\pi \approx -2.56 \in (-\pi, \pi]$

(+.5)

So $|2 - (10 - 4\pi)| = |-8 + 4\pi| = 4\pi - 8 \approx 4.56 > \pi \Rightarrow 2\pi - 4\pi + 8 = 8 - 2\pi$ ≈ 1.72

(b) [2] (WHW2 Ch3 #4) Recall on the line that B is *between* A and C if $AB + BC = AC$. Identify a point between the two points considered in (a) and verify the condition $AB + BC = AC$ is met.

If A & B have coord 2 and $10 - 4\pi$ resp.

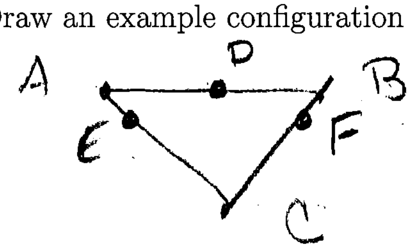
I think C with coord π could work :)

AB + BC (+1) [redacted] (+1)

$|2 - \pi| + |\pi - (10 - 4\pi)| = (\pi - 2) + (5\pi - 10) = \pi - 2 + 2\pi - 5\pi + 10 = 8 - 2\pi$ = AC

4. (Suggested 4.20) Let A, B, and C be distinct non-collinear points, and consider the segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Let D, E, and F be points so that $A - D - B$, $A - E - C$, and $B - F - C$.

(a) [2] Draw an example configuration of points that satisfies the above criteria.

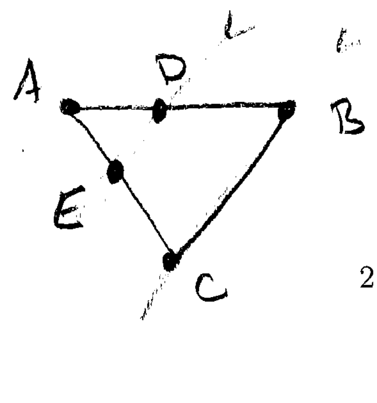


Distinct non collinear A, B, C (+1)
line segments (+.5)
Betweeness (+.5)

(b) [3] Is it possible to arrange D, E and F with the above constraints so that there is also a line l that passes through D and E but has no points between B and C? If so, sketch the possibility. If not, briefly explain why not.

yes? let $\overline{DE} \parallel \overline{BC}$ (+.5)

Sketch (+.5)



notation (+.5)
sketch (+1)
satisfy betweeness/etc (+.5)

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5. [4] (WHW2 Ch 2#29) Critique the following proof. Make sure to identify any logical problems if they exist!!

Theorem 1. *If ϕ is a fold with $\phi(A) = C$, $\phi(B) = D$, and $X \in \overline{AB}$, then $\phi(X) \in \overline{CD}$.*

start (1.5)
logic error (.1)
notation error (.1)
two changes (.1)
onto/since (1.8)

Proof. Use postulate L1 to assign coordinates 0, b, and x to points A, B, and X respectively with $b > 0$. Since $X \in \overline{AB}$, Theorem 2.4 allows us to conclude that the coordinates satisfy $0 < X < B$.

Notice also that Theorem 2.4 gives us that $C < \phi(X) < D$ implies $\phi(X) \in \overline{CD}$. Thus we will show that $C < \phi(X) < D$.

Recall Corollary 2.6A which showed that folds preserve the *between* relation, thus $0 < X < B$ implies that $\phi(0) < \phi(X) < \phi(B)$. We where given that assumptions that $\phi(A) = C$ and $\phi(B) = D$ thus we have $C < \phi(X) < D$ which is what we wanted to show. ☐

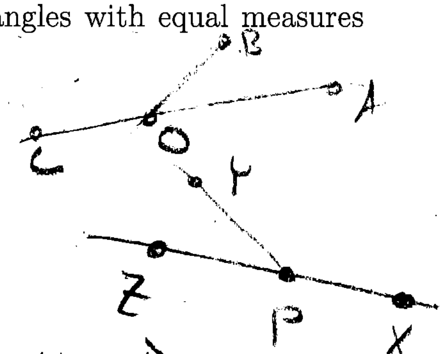
logic problem: The theorem needs to work for all rulers But we choose a specific one..., Unless we know how to get from 1 ruler to another, the theorem only holds when the coord of A is 0.

Notation problem: Points are being confused with their coordinates in all of the circled sections

start (1)
definitions (.1)
notation (.1)
Proof (1.1)
generalization (.1)

6. [8] (1/27 class) Theorem 4.17: Angles supplementary to angles with equal measures have equal measures. Let μ be a protractor.

Let $\angle AOB$ and $\angle BOC$ be supplementary
Let $\angle ZPY$ and $\angle YPX$ be the second set of supplementary angles with $\mu(\angle AOB) = \mu(\angle ZPY)$.



We want to show $\mu(\angle BOC) = \mu(\angle YPX)$

Since $\angle AOB$ and $\angle BOC$ are supplementary $\Rightarrow \mu(\angle AOB) + \mu(\angle BOC) = \pi$
thus $\Rightarrow \mu(\angle BOC) = \pi - \mu(\angle AOB)$.

Since $\mu(\angle AOB) = \mu(\angle ZPY)$ we have $\mu(\angle BOC) = \pi - \mu(\angle ZPY)$

However note that since $\angle ZPY + \angle YPX$ are supplementary

$\mu(\angle ZPY) + \mu(\angle YPX) = \pi$ or $\mu(\angle YPX) = \pi - \mu(\angle ZPY)$

By transitivity of the equal sign $\mu(\angle YPX) = \mu(\angle BOC)$
which is what we wanted to show. //

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Postulate 1. *Any two points determine a unique line.*

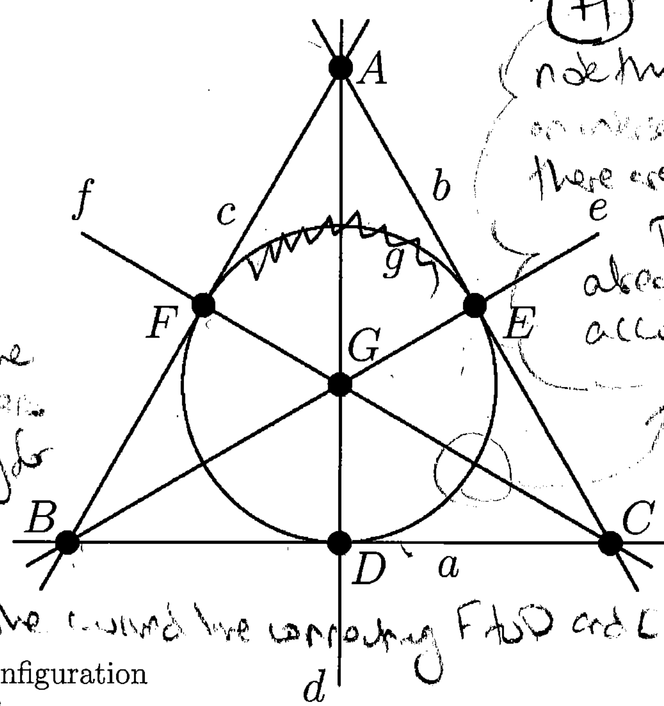
Postulate Finite 2. *Given any two distinct points on a line, there is a one-to-one correspondence, called a* ruler *between all points on the line with* $\{-1, 0, 1\}$ *that sends one of the two given points to 0, and the other to some number greater than 0. The number p assigned to a point P by the ruler is called the* coordinate.

Postulate 3. *Every line l determines a decompositions of the plane into 3 distinct sets:* H_0, H_1, *and l where: 1) Every pair of points in one of the* H_i *are on the same side of l, and 2) Every pair of points where one is in* H_0 *and the other is in* H_1 *are on opposite sides of l.*

7. Let us consider a finite set of points and lines known as the Fano Plane.of F_7: (+1)

note this is NOT an intersection b/c there are only 7 points already accounted for

There are seven points labeled with capital letters and seven lines labeled with lower case letters.



(a) [2] Does this configuration satisfy Postulate 1? Briefly justify your answer.

stated (1.5)

(1.5) [Yes? | Corners (ABC) the lines connecting them are clear. Similarly for middle (G).

(1)

| The points F, E, & D also satisfy with the curved line connecting F to D and E, etc

(b) [3] Is it possible for the above configuration to satisfy the Finite 2 postulate? If so, provide coordinates for points on line $\overline{DE}$. If not, explain why not.

stated (1.5)
set up 1-1 (+1)
between (+1.5)

The "line" g looks to violate between.
eg if coord of F, D & E are -1, 0, 1 resp.,
Looks like D-F-E but 0 ≯ -1 < 1 nor 0 > 1
If we removed curve between F & E would be OK

(c) [4] Does the above configuration satisfy Postulate 3? Justify your answer.

No?

stated (1.5)
def of H_i (+1)
eg that breaks (+1.5)

Consider the line g. The "plane" would be divided up into

$g = \{F, E, D\}$ $\quad H_0 = \{B, ?$

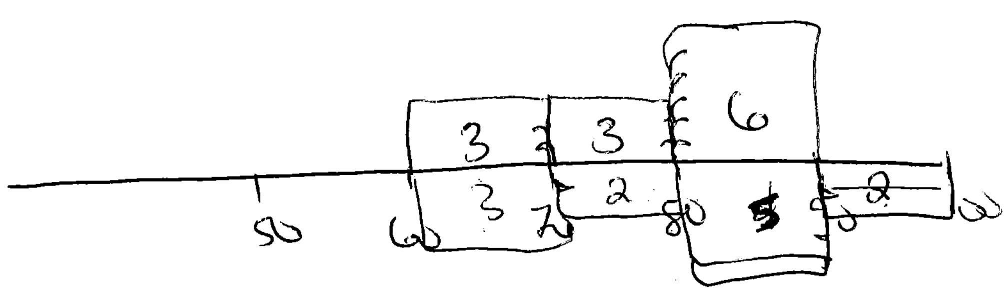
Notice C ∉ H_0 b/c $\overline{BC}$ intersects g (@ D) so C ∈ H_1

Notice A ∉ H_0 b/c $\overline{BA}$ intersects g (@ F) so A ∈ H_1 8

| Notice $\overline{AC}$ intersects g (@ E) so C and A cannot both be in H_1

$$\begin{array}{r} 25 \\ 20 \\ \hline 45 \end{array}$$

raw



actual 4/4 b/c
didn't like #7bc
parts

Median 79% 81
raw up