Exam 1 | # TMath 344 | Winter 2025

Show all your work. Reasonable supporting work must be shown to earn credit.

1. [10] Let A, B, C, and D be points with coordinates a, b, c, and d respectively. We use the notation from the text. Determine if the following make sense/*could* be true and be sure to justify your answer:

   (a) $a = b$

   (b) $A < b$

   (c) $a - b - c$

   (d) $\overline{AB} \in \overline{CD}$

   (e) $A \in CD$

2. Consider the statement: $\exists x \in \mathbb{Z}, x^2 = 2$.

   (a) [2] (2019Exam #2) Use as few symbols as possible to interpret the meaning with words.

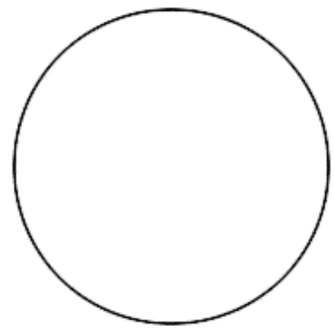
   (b) [3] (HW1 #1.13) Negate the above statement symbolically without using the $\exists$ symbol.

Definition 1. *If A and B are arbitrary points with coordinates a and b respectively where each is in the interval $(-\lambda, \lambda]$, then the* distance AB *from A to B is defined as:*

$$AB = \begin{cases} |a-b| & \textit{if } |a-b| \le \lambda \\ 2\lambda - |a-b| & \textit{if } |a-b| > \lambda \end{cases}$$

3. Consider the circle with circumference 2π so we let $\lambda = \pi$ in the distance defined above.

   (a) [3] (1dFolding Activity #4) Find the distance between the points with coordinates 2 and 10.



   (b) [2] (WHW2 Ch3 #4) Recall on the line that B is *between* A and C if $AB + BC = AC$. Identify a point between the two points considered in (a) and verify the condition $AB + BC = AC$ is met.

4. (Suggested 4.20) Let A, B, and C be distinct non-collinear points, and consider the segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Let D, E, and F be points so that $A - D - B$, $A - E - C$, and $B - F - C$.

   (a) [2] Draw an example configuration of points that satisfies the above criteria.

   (b) [3] Is it possible to arrange D, E and F with the above constraints so that there is also a line l that passes through D and E but has no points between B and C? If so, sketch the possibility. If not, briefly explain why not.

5. [4] (WHW2 Ch 2#29) Critique the following proof. Make sure to identify any logical problems if they exist!!

   Theorem 1. *If ϕ is a fold with $\phi(A) = C$, $\phi(B) = D$, and $X \in \overline{AB}$, then $\phi(X) \in \overline{CD}$.*

   Proof. Use postulate L1 to assign coordinates 0, b, and x to points A, B, and X respectively with $b > 0$. Since $X \in \overline{AB}$, Theorem 2.4 allows us to conclude that the coordinates satisfy $0 < X < B$.

   Notice also that Theorem 2.4 gives us that $C < \phi(X) < D$ implies $\phi(X) \in \overline{CD}$. Thus we will show that $C < \phi(X) < D$.

   Recall Corollary 2.6A which showed that folds preserve the *between* relation, thus $0 < X < B$ implies that $\phi(0) < \phi(X) < \phi(B)$. We where given that assumptions that $\phi(A) = C$ and $\phi(B) = D$ thus we have $C < \phi(X) < D$ which is what we wanted to show. □

6. [8] (1/27 class) Theorem 4.17: Angles supplementary to angles with equal measures have equal measures.

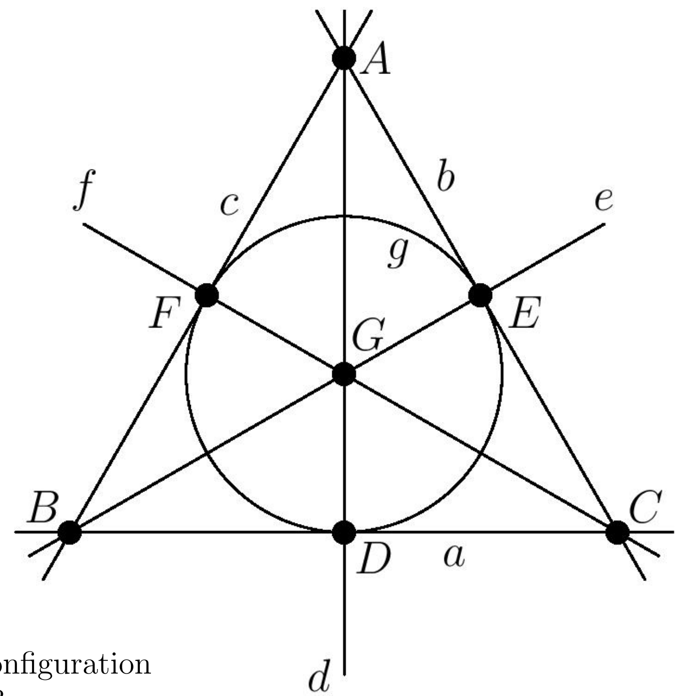
Postulate 1. *Any two points determine a unique line.*

Postulate Finite 2. *Given any two distinct points on a line, there is a one-to-one correspondence, called a* ruler *between all points on the line with* $\{-1, 0, 1\}$ *that sends one of the two given points to 0, and the other to some number greater than 0. The number* p *assigned to a point* P *by the ruler is called the* coordinate*.*

Postulate 3. *Every line* l *determines a decompositions of the plane into 3 distinct sets:* H_0*,* H_1*, and* l *where: 1) Every pair of points in one of the* H_i *are on the same side of* l*, and 2) Every pair of points where one is in* H_0 *and the other is in* H_1 *are on opposite sides of* l*.*

7. Let us consider a finite set of points and lines known as the Fano Plane.of F_7:

There are seven points labeled with capital letters and seven lines labeled with lower case letters.



(a) [2] Does this configuration satisfy Postulate 1? Briefly justify your answer.

(b) [3] Is it possible for the above configuration to satisfy the Finite 2 postulate? If so, provide coordinates for points on line $\overline{DE}$. If not, explain why not.

(c) [3] Does the above configuration satisfy Postulate 3? Justify your answer.