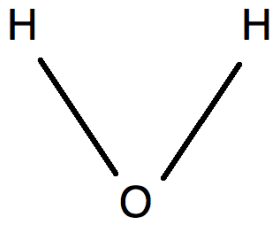
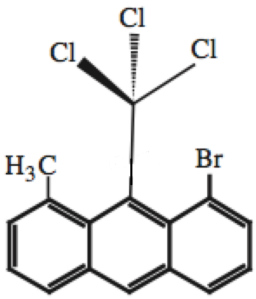
# Topological Symmetry Groups

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Given an abstract graph $\Gamma$, recall $Aut(\Gamma)$ is the collection (or abstract group!) or all permutations of the vertices that preserve adjacency. Find $Aut(\Gamma)$ for the abstract graphs (ignore the embedding information) below:



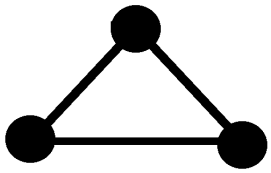

2. Assume all the above molecules are fairly rigid and embedded in $\mathbb{R}^3$ as shown above. Identify a permutation of the graph shown above that is not possible for the chemical to do in $\mathbb{R}^3$ with the given embedding.

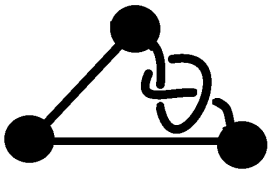
3. Do any of the graph permutations in problem 1 reverse orientation? If so, which ones?

4. Recall that the orientation preserving topological symmetry group ($\mathrm{TSG}_+$) of an embedded graph $\Gamma$ is a sub-collection (or abstract subgroup!) of $Aut(\Gamma)$ which are induced by an orientation preserving homeomorphism of the graph in $\mathbb{R}^3$. That is, there is a continuous map $h : \mathbb{R}^3 \to \mathbb{R}^3$ that has an inverse $h^{-1}$, and maps the graph back to itself. Note that homeomorphisms are continuous so images of straight lines can "bend" or "lengthen".

   Find the $\mathrm{TSG}_+$ of the two embeddings given in 1.

5. For each embedding $G$ of a graph $\Gamma$ below, find $Aut(\Gamma)$, $TSG(G)$, and $TSG_+(G)$.



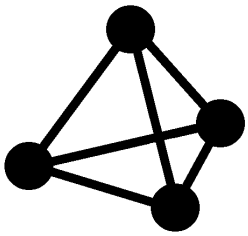


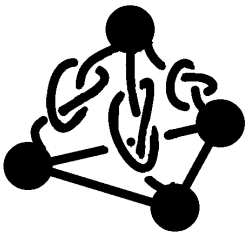
$Aut($  $)$ $\quad$ $Aut($  $)$

$TSG($  $)$ $\quad$ $TSG($  $)$

$TSG_+($  $)$ $\quad$ $TSG_+($  $)$

6. For each embedding $G$ of a graph $\Gamma$ below, find $Aut(\Gamma)$, $TSG(G)$, and $TSG_+(G)$.





$Aut($  $)$ $\quad$ $Aut($  $)$

$TSG($  $)$ $\quad$ $TSG($  $)$

$TSG_+($  $)$ $\quad$ $TSG_+($  $)$