

Prove or disprove the following:

1. The sum of an irrational number and a rational number is irrational.
2. If m and n are integers and mn is even, then m is even or n is even.
3. If n is a perfect square, then $n + 2$ is not a perfect square.
4. At least ten of any 64 days chosen must fall on the same day of the week.
5. The $\sqrt{2}$ is irrational.
6. We can tile a standard checkerboard with the upper left and lower right corner squared deleted with dominoes (where each domino covers two squares on the checkerboard).
7. The number $5x + 5y$ is an odd integer when x and y are integers of opposite parity.
8. Either $2 \cdot 10^{400} + 10$ or $2 \cdot 10^{400} + 11$ is not a perfect square.
9. There are no integer solutions of x and y to the equation $2x^2 + 5y^2 = 15$.
10. There is no positive integer n such that $n^2 + n^3 = 100$.
11. Every positive integer is the sum of 36 fifth powers of nonnegative integers
12. Assume the truth of the theorem: $\sqrt[n]{n}$ is irrational whenever n is a positive integer that is not a perfect square. Is $\sqrt{2} + \sqrt{3}$ irrational or not?