

# Relations

$R_1$  is a relation on  $\mathbb{Z}$   
 defined by  
 $\{(a, b) \mid a + b \leq 3\}$

$R_2$  is a relation on  $A = \{1, 2, 3, 4\}$   
 defined by  
 $\{(a, b) \mid \max(a, b) = b\}$

$R_3$  is a relation on  $\mathbb{Z}$   
 defined by  
 $\{(a, b) \mid a = b^2\}$

$R_4$  is a relation on  $\mathbb{R}$   
 defined by  
 $\{(a, b) \mid a \cdot b \geq 0\}$

- For each of the relations  $R_i$  defined above, list five ordered pairs that are in the relation.

$R_1$ :

$R_2$ :

$R_3$ :

$R_4$ :

- For each of the relations  $R_i$  above, determine if  $R_i$  has the properties listed (if applicable). If the relation does not have the indicated property, identify an ordered pair (or set of ordered pairs) that exhibits the failure.

	$R_1$	$R_2$	$R_3$	$R_4$
reflexive				
symmetric				
antisymmetric				
transitive				

3. Find an example of a relation on a set that is both symmetric and antisymmetric.

4. Let  $R$  be the relation on the set  $\{1, 2, 3, 4, 5\}$  containing the ordered pairs:  
 $(1, 1)$ ,  $(1, 2)$ ,  $(1, 3)$ ,  $(2, 3)$ ,  $(2, 4)$ ,  $(3, 1)$ ,  $(3, 4)$ ,  $(3, 5)$ ,  $(4, 2)$ ,  $(4, 5)$ ,  $(5, 1)$ ,  $(5, 2)$ , and  
 $(5, 4)$ .

(a) Find  $R \circ R$

(b) Find  $R^2$