

Prove or disprove the following:

1. Whenever  $n$  is a positive integer, 3 divides  $n^3 + 2n$ .
2. Let  $n$  be a positive integer greater than or equal to two. If  $n$  people stand in a line with the first person in line a woman and the last person in line a man, then somewhere in the line there is a woman directly in front of a man.
3. Consider an arithmetic progression  $a + (a + d) + (a + 2d) + \dots + (a + nd)$ . For all positive integers  $n$  we have, 
$$\sum_{i=0}^n (a + id) = \frac{(n + 1)(2a + nd)}{2}.$$

4. Suppose that  $a_j \equiv b_j \pmod{m}$  for  $j = 1, 2, \dots, n$ , then 
$$\sum_{j=1}^n a_j \equiv \sum_{j=1}^n b_j \pmod{m}.$$

5. The algorithm to the right evaluates all strings of length  $n$  as “True” if they are palindromes and false otherwise.

```
def Recurse(x): #x is a list type
    if len(x)<2:
        return True
    else:
        if x[0]==x[len(x)-1]:
            del x[len(x)-1]
            del x[0]
            return Recurse(x)
        else:
            return False

x = [1,2,1]
Recurse(x)
True

y=[1,2,3,4]
Recurse(y)
False
```

6. Let  $n$  be a positive integer and  $x$  be a real number. There is no recursive algorithm that can compute  $x^n$ .
7. Let  $s$  be a string with  $n$  characters. There is no recursive algorithm that can reverse the characters in the string  $s$ .
8. 64 divides  $3^{2n+2} + 56n + 55$  for every positive integers  $n$ .
9. For all positive integers,  $a - b$  is a factor of  $a^n - b^n$ .
10. There is no recursive algorithm for computing the greatest common divisor of two positive integers.