Extra Credit

Proof Practice take 2

TCSS 321

Prove or disprove the following:

1. For all positive integers n, $\sum_{i=1}^{n}(i^3) = \left(\frac{n(n+1)}{2}\right)^2$
2. The number $5x + 5y$ is an odd integer when x and y are integers of opposite parity.
3. Let S be the set that contains a set x if the set x does not belong to itself, so $S = \{x | x \notin x\}$. S is a member of S.
4. Let S be the set that contains a set x if the set x does not belong to itself, so $S = \{x | x \notin x\}$. S is a not a member of S.
5. 2 divides $n^2 + n$ whenever n is a positive integer.
6. If A, B, and C are sets, then $\overline{A} \cap \overline{B} \cap \overline{C} = \emptyset$.
7. If n is an integer greater than 4, then $2^n > n^2$.
8. For a positive integer n, n is even if and only if $7n + 4$ is even.
9. Consider a rectangular checkerboard that you'd like to divide into its constituent squares. You can either make (complete) cuts along the vertical lines or horizontal lines separating the squares. There is no formula to determine how many cuts you must make to break the checkerboard into the n separate squares.
10. If A, B, and C are sets then:
    $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.
11. For all nonnegative integers n, $\sum_{i=0}^{n}\left(\frac{-1}{2}\right)^i = \frac{2^{n+1} + (-1)^n}{3 \cdot 2^n}$.
12. If n is an integer greater than 6, then $3^n < n!$.