

Prove or disprove the following:

1. The sum of an irrational number and a rational number is irrational.
2. Let  $m$  and  $n$  be integers. If  $mn$  is even, then  $m$  is even or  $n$  is even.
3. 2 divides  $n^2 + n$  whenever  $n$  is a positive integer.
4. If  $n$  is a perfect square, then  $n + 2$  is not a perfect square.
5. If  $n$  is an integer greater than 4, then  $2^n > n^2$ .
6. At least ten of any 64 days chosen must fall on the same day of the week.
7. The  $\sqrt{2}$  is irrational.
8. We can tile a standard checkerboard with the upper left and lower right corner squared deleted with dominoes (where each domino covers two squares on the checkerboard).
9. The number  $5x + 5y$  is an odd integer when  $x$  and  $y$  are integers of opposite parity.
10. Either  $2 \cdot 10^{400} + 10$  or  $2 \cdot 10^{400} + 11$  is not a perfect square.
11. There are no integer solutions of  $x$  and  $y$  to the equation  $2x^2 + 5y^2 = 15$ .
12. There is no positive integer  $n$  such that  $n^2 + n^3 = 100$ .
13. Every positive integer is the sum of 36 fifth powers of nonnegative integers
14. Assume the truth of the theorem:  $\sqrt{n}$  is irrational whenever  $n$  is a positive integer that is not a perfect square. Is  $\sqrt{2} + \sqrt{3}$  irrational or not?
15. Dominoes cannot tile a standard checkerboard with the four corners removed.
16. A checker board with the same number of white squares as black, is tillable by dominoes.