Somemore Logic

with quantifiers!

Let p, q, and r be propositions for the entirety of this worksheet (front & back).

1. Below are several ways to combine (using textbook notation) $\neg$, $\wedge$, and $\vee$.

   (a) Find the truth tables for each by hand or by using Sage.

(i) $\neg(p \wedge q)$ (ii) $(\neg p) \wedge q$ (iii) $p \wedge (\neg q)$ (iv) $(\neg p) \wedge (\neg q)$

(v) $\neg(p \vee q)$ (vi) $(\neg p) \vee q$ (vii) $p \vee (\neg q)$ (viii) $(\neg p) \vee (\neg q)$

   (b) Do any of the above truth tables look the same (i.e. are there any logical equivalences)? Which ones?

2. Perform the same investigation on the following (using textbook notation) and try to identify another logical equivalence.

(i) $p \vee (q \wedge r)$ (ii) $p \wedge (q \vee r)$ (iii) $(p \vee q) \wedge (p \vee r)$ (iv) $(p \wedge q) \vee (p \wedge r)$

Let x be an integer between -3 and 3.

3. Translate the following symbolic propositions into English sentences and determine the truth value.

   (a) $\forall x, x + 1 > x$.

   (b) $\exists x, x + 1 > x$.

   (c) $\forall x < 0, x^2 < 0$.

   (d) $\forall x, (x < 0 \rightarrow x^2 < 0)$

   (e) $\exists x > 0, x^2 = 2$.

   (f) $\exists x, (x > 0 \land x^2 = 2)$.

   (g) $\neg \forall x, x^2 > x$.

   (h) $\exists x, \neg(x^2 > x)$.

4. Do any of the above statements seem logically equivalent? Which ones?