

Cardinality

the infinite kind

David Hilbert was a great mathematician (whose bio is on page 171) who single handedly steered 100 years of mathematical work by asking 23 questions in 1900.

But more to the point of the worksheet, Hilbert was known for a thought experiment involving a hotel with a countably infinite number of rooms (a theater in Futurama references this idea). Assume Hilbert owns a hotel with a countable number of rooms, all of which are occupied for the remainder of this worksheet, and answer the following:

1. How many rooms are in the hotel?
2. If one person shows up at Hilbert's door asking for a room, how could Hilbert accommodate him without throwing out any of his current guests? Describe the steps that Hilbert takes algorithmically.
3. If 1,000 people show up at Hilbert's door asking for rooms, how could Hilbert accommodate him without throwing out any of his current guests? Describe the steps that Hilbert takes algorithmically.
4. Say a set of people arrive at Hilbert's hotel (which is again full) and each would like their own room. Each person is uniquely identified by a positive integer. Can Hilbert accommodate the new set of people without turning any of his current guests away?