Key

Midterm TMath 171 Winter 2026

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and one 1-sided 8.5 in by 11 in sheet of notes.

1. [6] Let a, b, and c be whole numbers. Are the following statement always true, sometimes true, or never true? Briefly justify your answer.

   (a) (§3.4 #34) $a^c \times a^b = a^{cb}$

   start (1.5) expdef (+1) notation/sense (1.5)

   Sometimes true (+1)

   eg $5^0 \times 5^0 = 1 \times 1 = 1 = 5^{0\times 0}$ (true)

   eg $2^2 \times 2^3 = 4 \times 8 = 32 \neq 64 = 2^6 = 2^{2\times 3}$

   (b) (ExtraPractice3.2 #17) $a - b = b - a$

   start (1.5) subtraction (+1) notation/sense (1.5)

   Sometimes true (+1)

   eg $2 - 2 = 0 = 2 - 2$ (if $a = b$)

   eg $3 - 2 = 1 \neq -1 = 2 - 3$

2. At Joe's Cafe 1 cup of coffee and 3 doughnuts cost \$6.60, and 2 cups of coffee and 2 doughnuts cost \$7.20.

   (a) [1] (Quiz1#2) What kind of strategy (eg, drawing, tables, clever guessing & checking, models, work backwards, etc) might you try to figure out the cost of 1 cup of coffee?

   Model perhaps? # of coffee = c and # of doughnuts = d

   OR

   (b) [3] (ExtraPractice §1.1#23) Use your strategy (and others if you need it) to find the cost of 1 cup of coffee.

   understand prob (1.5) start strategy (1.5) used strategy (1.5) notation/sense (1.5) legal steps (1.5)

   (α) $1c + 3d = \$6.60$

   (β) $2c + 2d = \$7.20$

   If we use (α) & write $1c = 6.60 - 3d$ we could substitute in $6.60 - 3d$ for $1c$ in equation (β): $2[6.60 - 3d] + 2d = 7.20$

   Then algebraically solve for d

   $13.2 - 6d + 2d = 7.20 \Rightarrow -4d = -6$

   $\Rightarrow d = \frac{-6}{-4}$ or \$1.50. Since $c = 6.60 - 3d = \$2.10$

   OR

   If $2c + 2d = 7.20$

   then $1c + 1d = \frac{1}{2}(7.20)$

   or $1c + 1d = 3.60$

   or $1c = 3.60 - 1d$

   we could use this in the first fact that $1c + 3d = 6.60$

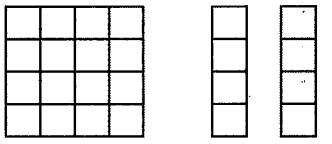
   $3.60 - 1d + 3d = 6.60$

   so $2d = 3.00$

   $\Rightarrow d = \$1.50$. Since $c = 3.60 - 1d$, $c = \$2.10$

3. Consider the number in the base pieces below with 1 flat, 2 longs, and 0 units.

   (a) [2] (Quiz2#2) Write the number of units in positional notation for the given base.



+1.5 4 units in long => base four

+1.5

120_{four}

   (b) [2] (NumberSystemActivity #3) Determine the total number of units, reporting in the Hindu-Arabic number system.

start +1.5

1 flat + 2 longs

+1.5 +1.5

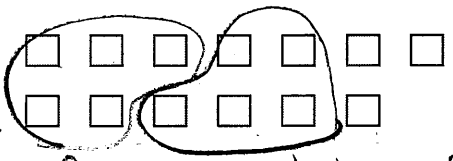
$4 \times 4 + 2 \times 4$

+1.5

$16 + 8 = 24$

4. (§3.1 #12) Consider the number of units shown below.

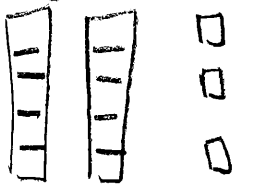
   (a) [2] Sketch the minimum number of base pieces for base five to represent the set of units shown.



Can group 10 into 2 longs and have 3 units left over

grouping in fives +1.5

got it +1.5



+1.5 +1.5

   (b) [1] Write the number of units in positional notation for base five.

23_{five}

   (c) [1] Write the number of units in a different number system of your choosing (eg. Egyptian, Babylonian, etc)

Hindu-Arabic: $2 \times 5 + 3 = 13$ | Egyptian: ∩||| | Babylonian: ‹ FT | Base 4: 31_{four}

5. [4] (AddActivity #4) An equation is given below in base five. Replace the suit symbols with the digits 2, 3, and 4 to make a true equation.

+1.5 +1.5

$1\clubsuit^{3}_{five} + \heartsuit 3_{five} = \spadesuit 1_{five}$

Step 2

The units create 1 long so we write

1 long + ♡ long = ♠ long

2 long + ♡ long = 4 long

b

=> ♡ = 2 and ♠ = 4

+.5 notation

+1 units/ ♣

+1.5 regrouping

+1 understand ♣ can't stay in base five

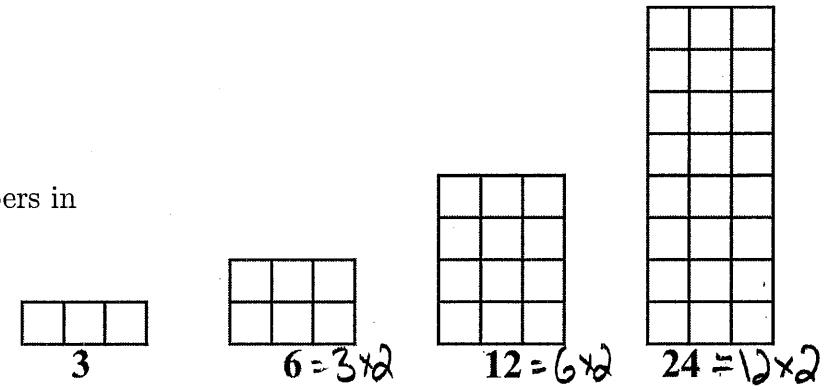
Step 1

regrouping must occur for the units to return to 1 digit

try ♣ = 2, 2 + 3 = 10_{five} ✗

try ♣ = 3, 3 + 3 = 11_{five} ✓

6. (PatternActivity#4) Consider the sequence of numbers illustrated below.



(a) [3] Find the next two numbers in the sequence.

pattern of mult by 2 (+1)
examining steps between figures (+1)

24×2 = 48 (+1.5) and 48×2 = 96 (+1.5)

(b) [2] Identify if the sequence is recursive, arithmetic, geometric, or none of the above. Justify your answer.

def (+1) To get from one number to the next we repeatedly multiply by a fixed ratio of 2.

(c) [2] Write down an expression for the 20th number in the sequence.

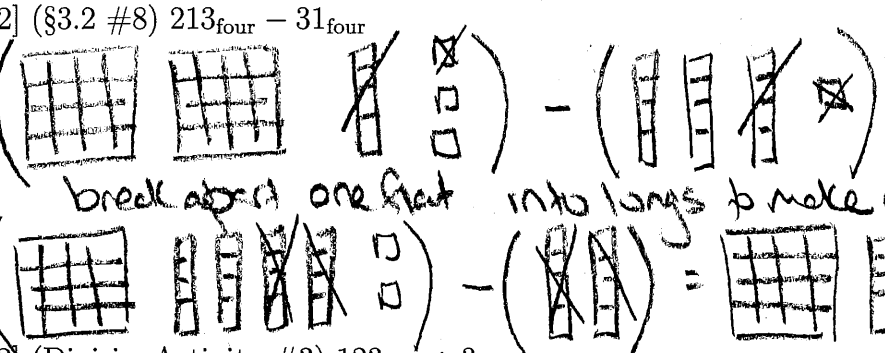
(+1.5) $3 \times 2 \times 2 \times 2 \times \cdots \times 2$ (+1.5) or $3 \cdot 2^{19}$ or 1,572,864

(+1) {19 of these — got it +1.5

7. Show work and compute (you do not need to explain it to a 4th grader):

(a) [2] (§3.2 #8) $213_{four} - 31_{four}$

regrouped 1 flat (+1.5)
units (+1.5)
flats (+1.5)
longs (+1.5)



$= 122_{four}$

OR

$$\begin{array}{r} {}^{1}{}^{5} \\ 213_{four} \\ -\ 31_{four} \\ \hline 122_{four} \end{array}$$

— or —

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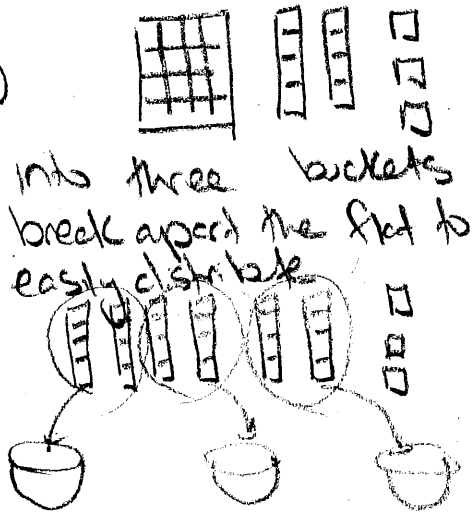
(b) [2] (DivisionActivity #3) $123_{four} \div 3_{four}$

division (+1.5)
work (+1)
got it (+1.5)



2 longs and 1 unit into each bucket

so 21_{four}

or 9

8. [3] Find a number that:

- (+1) • is not written in base 10,
- (+1) • has 3 digits, and
- (+1) • is less than 50 units.

There are LOTS of answers for this?

101_{two} (or 4+1=5)

9. Grade the work that follows. The work may be correct or incorrect. If correct, briefly justify why. If incorrect, find the error(s) & try to detect the reason for the error. You do *not* need to provide the correct computation if there is an error.

(a) [3] (§3.2 #22)

broke up units & longs indep

$(95+37=(5+7)+(90+30))$ (+1) Correct

(+1) Why (+1) understand what student is doing

$$\begin{array}{r} 95 \\ +\ 37 \\ \hline 12 \\ +\ 120 \\ \hline 132 \end{array}$$

adding the units together with appropriate regrouping

adding the longs together with appropriate regrouping

added the flats, longs and units together at the end.

(b) [3] (ExtraPractice §3.3 #9)

(+1) understand what student did (+1) Correct

(+1) Why essentially distribution with $(20+4)\times(30+8)$

$$\begin{array}{r} 24 \\ \times\ 38 \\ \hline 32 \\ 160 \\ 120 \\ +\ 600 \\ \hline 912 \end{array}$$

← mult the units 8×4

← mult long 3 by unit 4 so 30×4

← mult unit 8 by long 2 so 8×20

← mult the longs 3 and 2 by each other so 30×20

$= 20\times30 + 4\times30 + 20\times8 + 4\times8$

(c) [3] (Quiz3 #3)

$$6^2 - 2^4 + 5 = 36 - 16 + 5 = 36 - 21 = 15$$

(+1) incorrect

(+1) problem 16+5 was added but

(+1) subtraction 36-16 should be done first

reason: likely PEMDAS and student think "A" means addition should always be done before subtraction

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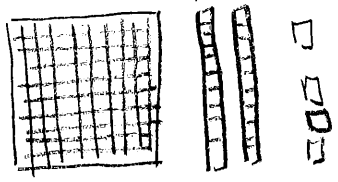
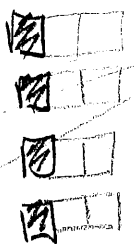
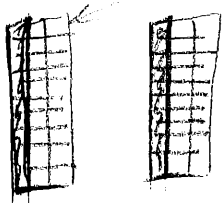
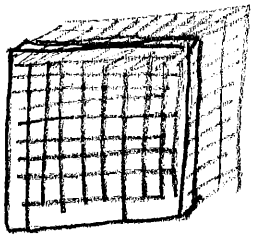
10. [5] (§3.3 #6) Introduce how to multiply numbers as you would to an elementary school student who had forgotten what multiplication is. Provide a conceptual explanation and not just a procedural explanation. Use the example 124×3 to help illustrate.

Multiplication is like a stretch.
Lets look at 124×3. We'll take 124 and stretch it by 3.

Recall 124 is 1 flat, 2 longs or 4 units so

we are going to stretch this # by 3.
Lets do the stretching in pieces.

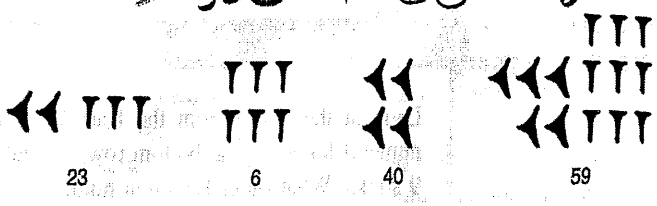
Units, stretch by three: the original:

← stretched parts
=> 12 units
or 1 long & 2 units

longs stretch by three: the original

=> 6 longs

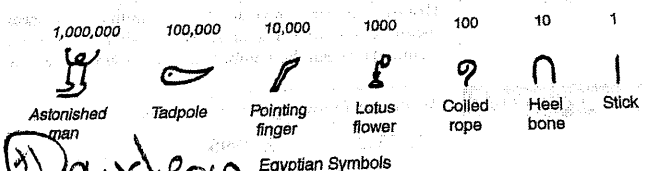
flat stretched by three: original is closest to us & stretched part extends into 3D

=> 3 flats

Then we add these parts together so

$$124 \times 3 = (100 + 20 + 4) \times 3 = 100 \times 3 + 20 \times 3 + 4 \times 3$$
$$= 300 + 60 + 12 = 372$$



Babylonian



Egyptian Symbols

(+1) awkward
(+1) regroup
(+1) conceptual
(+1) procedure
(+1) eg is 372

5

$$\begin{array}{r} 22 \\ 23 \\ \times\ \ 5 \\ \hline 50 \end{array}$$