Key

Midterm **TMath 171** Autumn 2025

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and one 1-sided 8.5 in by 11 in sheet of notes. Show your work as you would for a *colleague* unless otherwise indicated.

1. [3] (MultActivity #2) Let a, b, and c be whole numbers. Determine if following statement is always true, sometimes true, or never true? Briefly justify your answer.

Start (1.5) (1.5)

$$a + (b \times c) = (a \times b) + (a \times c)$$

Sometimes true (1.5)

eg if $a=b=c=1$

$a+(b\times c)=2$ and

$(a\times b)+(a\times c)=2$.

(1.5) Sometimes false

eg $a=0$ and $b=c=1$

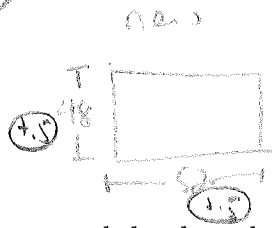
$a+(b\times c)=0+1=1$ but

$(a\times b)+(a\times c)=0+0=0$.

2. (Quiz1 #2) Suppose you own a square piece of land. You trade your land for a rectangular lot. The length of your new lot is 2 meters longer than the original lot and the width is 2 meters shorter than the original lot.

   (a) [2] If the original land is 50 meters by 50 meters, what are the dimensions of the new lot?

orginal (1.5)



(1.5)

(1.5)

So 18m by 52 m

Start (1.5)

   (b) [2] Does the new lot have as much land as the old? Provide some justification.

orginal area:

$50\times50 = 2500\ m^2$ (1.5)

new lot area:

$18\times52 = 2496\ m^2$ (1.5)

So less land (1.5)

area (1.5)

   (c) [3] Do your conclusions above generalize for any square piece of land? Provide some justification.

notation (1.5)

define terms/variables (+1)

generalizing (+1)

1.5 If another ex

(1.5) yes?

Assume the orginal land is x meters by x meters.

The new land would by $(x-2)$ meters by $(x+2)$ meters.

The area of orginal land: $x\cdot x$ or x^2

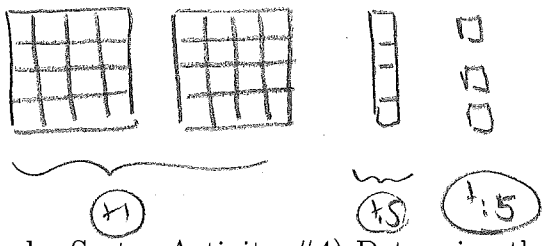
area of new land: $(x+2)(x-2) = x^2+2x-2x-4 = x^2-4$

So always $4\ m^2$ less

10

3. Consider the number written in base 4 with 2 flats, 1 long, and 3 units or 213_{four}.

   (a) [2] (§3.2#8) Sketch the base pieces for 213_{four}.



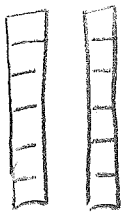
   (b) [2] (NumberSystemActivity #4) Determine the total number of units in 213_{four}, reporting in the Hindu-Arabic number system.

Hindu-Arabic the number system we've been taught.

$2 \cdot 4^2 + 4^1 + 3 = 2 \cdot 16 + 7 = 32 + 7 = 39$

4. (ExtraPractice §3.1 #11) Consider the number of units shown below.

   (a) [2] Sketch the minimum number of base pieces for base six to represent the set of units shown.



   (b) [1] Write the number of units in positional notation for base six.

20_{six}

5. [3] Is Egyptian or Babylonian a positional number system? Provide justification for your answer.

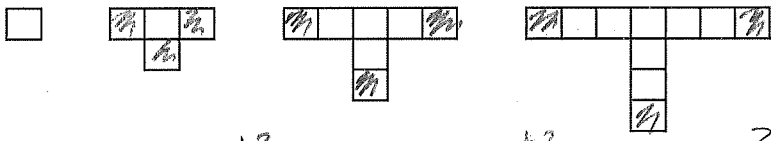
Babylonian is? Each position is a power of 60,

Egyptian is not? Cause 9 ∩ is equal to ∩9 or 110.

$\overset{2}{59}$
$\times\ 3$
177

/// new boxes from one step to next

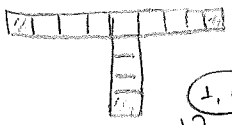
6. (ProblemSolvActivity #1) Consider the sequence of numbers illustrated below.

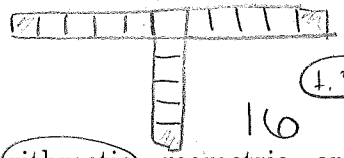


(a) [3] Find the next two numbers in the sequence.

start (+.5)
notation/sense (+.5)

1 +3 4 +3 7 +3 10 } (+1)

so  (+.5) 13

and  (+.5) 16

(b) [2] Identify if the sequence is arithmetic, geometric, or neither. Justify your answer.

The difference between each term is a fixed #3. (+1.5)

(+1) def of arithmetic/geo sense (+.5)

(c) [2] Find the 60th number in the sequence.

start (+.5)

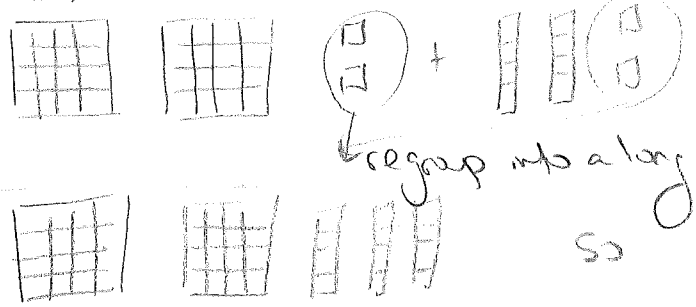
seq #	1	2	3	4	5	6	n
	1	1+3	1+6	1+9	1+12	1+15	1+3(n-1)

(+1) (added 3, fifty nine times)

so $1+3(59)$ (+.5)
$1+177 = 178$

7. Show work & compute (your audience is a colleague but still highlight any regrouping!):

(a) [2] (Quiz3 #1) $202_{\text{four}} + 22_{\text{four}}$

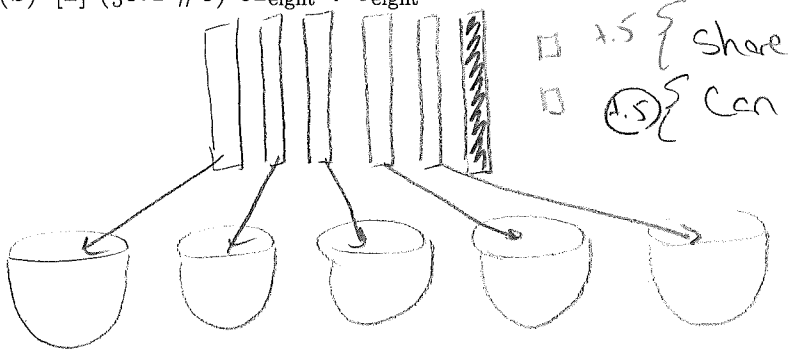


notation (+.5)

regroup into a long } (+1)

so 230_{four} (or 44) (+.5)

(b) [2] (§3.4 #8) $62_{\text{eight}} \div 5_{\text{eight}}$



(+.5) { share across 5 buckets
(+.5) { can give each bucket 1 long

We'll need to break up the remaining long into units to easily share the rest

} the units to share. (+1)

We can share 2 units across the 5 buckets
so our answer is 12_{eight} (or 10) (+.5)

8. [3] Find a pattern for a sequence of numbers that
   - form a geometric sequence, and
   - begins with 3.

LOTS of answers work here! 3, 6, 12, 24, 48, (repeatedly multiply) notation +1

9. Grade the work that follows. The work may be correct or incorrect. If correct, briefly justify why. If incorrect, find the error(s) & try to detect the reason for the error.

(a) [3] (AddActivity #1) Not Correct +.5

start +.5 sense +.5

$$\begin{array}{r} 42_{five} \\ +\ 34_{five} \\ \hline 76_{five} \end{array}$$ +1

1.5 trade

The student forgot to regroup twice!

$2_{five} + 4_{five} =$  $= 11_{five}$

This means we need a "carry" to the longs position.

Similarly five longs can be grouped to a flat

(b) [3] (§3.2 #8)

start +1.5 sense +1.5 subtraction +1.5

$$52 - 14 \Rightarrow 50 - 10 + 2 - 4 \Rightarrow 40 - 2 \Rightarrow 38$$

+3

Correct +1.5

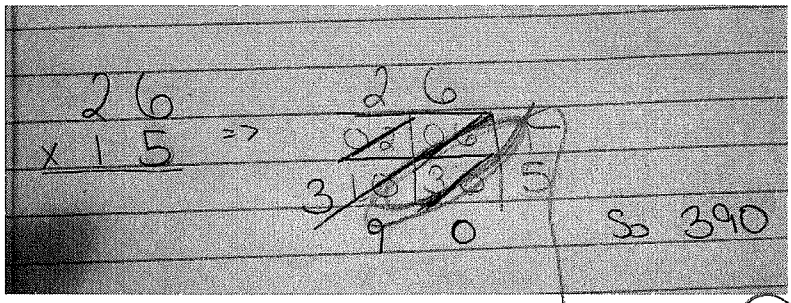
Student considered the tens/longs (50-10) then the units (2-4).

The tens & units where then joined back together.

(c) [3] (SeveralAlgorithms)

start +.5 sense +.5 multiplication +1.5

$$26 \times 15 \Rightarrow$$



+1

Correct +1.5

The lattice method highlights the different positions & add the correct ones together.

eg the longs/tens are so 6 tens + 3 tens or 9 tens in the final answer.

12

10. Consider $45 - (3 \div 3) \times 4$

(a) [1] (Quiz3 #2) Circle the operation above that should be performed first:

(b) [1] How would you modify the above expression to make it more clear the order of the operations?

$45 - [(3 \div 3) \times 4]$

added parenthesis (.5)
in correct spot (.5)

11. [5] (ExtraPractice §3.4 #33) Introduce how exponents and multiplication interact with each other as you would to an elementary school student who had forgotten. Use the example $5^4 \times 5^6$ in the explanation.

start (.5)

audience (+1)
notation/sense (+1)
got it (.5)

(+1) Remember that exponents are repeated addition so that was already there!)

note, stating a rule will not help a student remember

$5^4 = 5 \times 5 \times 5 \times 5$.

We'll use this to consider $5^4 \times 5^6$.

Since $5^4 = \underbrace{5 \times 5 \times 5 \times 5}_{4 \text{ times}}$ and $5^6 = \underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}_{6 \text{ times}}$

then $5^4 \times 5^6 = (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5 \times 5)$ (+1)

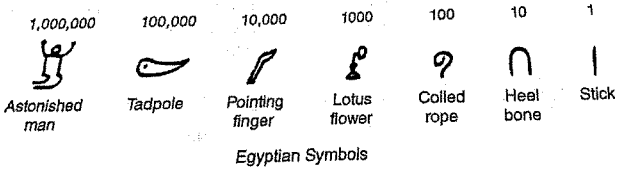
or 5 times itself 10 times :)

(↳ using associativity)

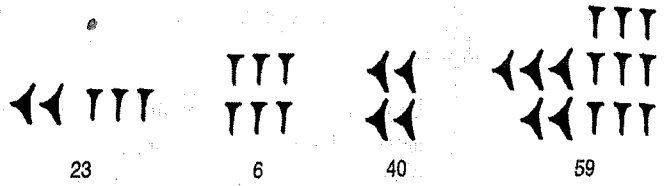
so 5^{10}.

This generalizes in fact? For a base b raised to whole # exponents n and m,

$b^n \times b^m = b^{n+m}$.



Egyptian Symbols



Babylonian

7

20
23
7

50