

The class before the exam there will be a chance to earn extra credit. Groups of two to three can present a solution to one of the problems below. Up to 4% can be earned:

- [1] Mastery of the problem: Do you understand the problem and all of the steps used to solve it? Would you be able to solve the problem if given a slightly different question?
- [1] Presentation of the problem: You are presenting material to your classmates that will be on their exam next week! Take care to explain your steps and why you take them but your group also needs to complete your presentation in under 10 minutes!
- [1] Presentation: Do you interact with the class? Do you make eye contact?
- [1] Fielding questions: Can you understand the questions and give a cohesive answer?

## Word Problem Practice

1. A business makes three sizes of widgets: small, medium, and large. it costs \$2.50 to make the small widget, \$4.00 for a medium widget, and \$4.50 for a large widget. The fixed cost for the business are \$8000. Express the cost of making  $x$  small widgets,  $y$  medium widgets, and  $z$  large widgets as a function of three variables  $C = f(x, y, z)$ . Find  $f(3000, 5000, 4000)$  and interpret it. Note there are no returns of scale here. Modify your function  $C$  to make this problem more realistic.

2. A function  $f$  of two variables is known to be continuous and provide the values in specified to the right.

$y \setminus x$	1.0	1.1	1.2
2.0	5	7	10
2.2	4	6	8
2.4	3	5	6

Your boss would like you to develop a linear model that could be used to estimate the value of  $f(1.4, 2.35)$ . Build the model and justify the choices/steps that you make.

3. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation  $PV = 8.31T$  where  $P$  is measured in kilopascals,  $V$  in liters, and  $T$  in kelvins. Use linear approximations to find the approximate changes in the pressure if the volume increases from 12L to 12.2L and the temperature decreases from 310K to 308K. Interpret your answer and provide units.

4. If  $R$  is the total resistance of three resistors, connected in parallel, with resistances  $R_1$ ,  $R_2$ , and  $R_3$ , then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

If the resistances are measured in ohms as  $R_1 = 25\Omega$ ,  $R_2 = 40\Omega$ , and  $R_3 = 50\Omega$  with a possible error of .5% in each case, estimate the maximum error in the calculated value of  $R$ .

5. A model for the surface area of a human body is given by  $S = 0.1091w^{0.425}h^{0.725}$ , where  $w$  is the weight (in pounds),  $h$  is the heights (in inches), and  $S$  is measured in square feet. This is handy in D&D if you happen to be a wizard and want to tattoo spells onto your skin.... If the errors in measurement for  $w$  and  $h$  are at most 2%, use a linear approximation to estimate the maximum percentage error in the calculated surface area. Interpret your answer and provide units.

6. Suppose over a certain region of space that the electrical potential  $V$  is given by  $V(x, y, z) = 5x^2 - 3xy + xyz$ . Find the rate of change of the potential at  $P(3, 4, 5)$ . in the direction of  $\vec{i} + \vec{j} - \vec{k}$ . In which direction does  $V$  change the most rapidly at  $P$ ?
7. If the length of the diagonal of a rectangular box must be  $L$ , what is the largest possible volume? Justify your answer.
8. The temperature  $T$  in a metal ball is inversely proportional to the distance from the center of the ball. If we let the center of the ball be the origin then the temperature at the point  $(1, 2, 2)$  is  $120^\circ$ . Find the rate of change of  $T$  at  $(1, 2, 2)$  in the direction toward the point  $(2, 1, 3)$ .
9. A grain silo is to be built by attaching a hemispherical roof and a flat floor onto a circular cylinder. Use Lagrange multipliers to show that for a total surface area  $S$ , the volume of the silo is maximized when the radius and height of the cylinder are equal.
10. Section 15.1 #6
11. Section 15.1 #7