Taylor Series Errors

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Taylor's Inequality: If f is a differentiable function through order $n+1$, then the error, $R_n(x)$, between f and the n^{th} Taylor approximation centered at c is:

$$|R_n(x)| \leq \frac{1}{(n+1)!}(x-c)^{n+1} \max |f^{(n+1)}(z)|$$

where z is a value between x and c.

1. Consider $y = \sin(x)$ and $T_4(x)$ where $T_4(x)$ is centered at $\frac{\pi}{2}$.

   (a) Find an upper bound for the error between $y = \sin(x)$ and $T_4(x)$ where x is between $\frac{\pi}{3}$ and $\frac{3\pi}{2}$.

   (b) Find an upper bound for the error between $y = \sin(x)$ and $T_4(x)$ when $x = \frac{\pi}{3}$.

   (c) Did you need to find $T_4(x)$ to answer the above question?

2. How many terms/what degree do we need for $T_n(x)$ centered at 0 to approximate $\cos(.1)$ with error less than .001?

3. We take measurements at discrete times and collect the data in the table below. The phenomenon has a well understood third derivative of N which has been graphed below. Use this information to find an upper bound for for $T_2(14)$.

t	9	10	11	12	13
$N(t)$	55	57	58	60	64