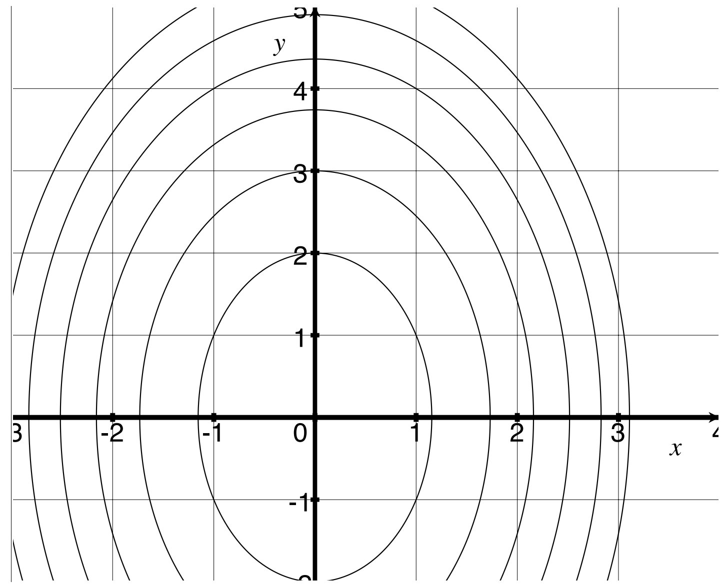
Derivatives with Direction Review

1. Let $h(x, y) = 5000 - 30x^2 - 10y^2$.

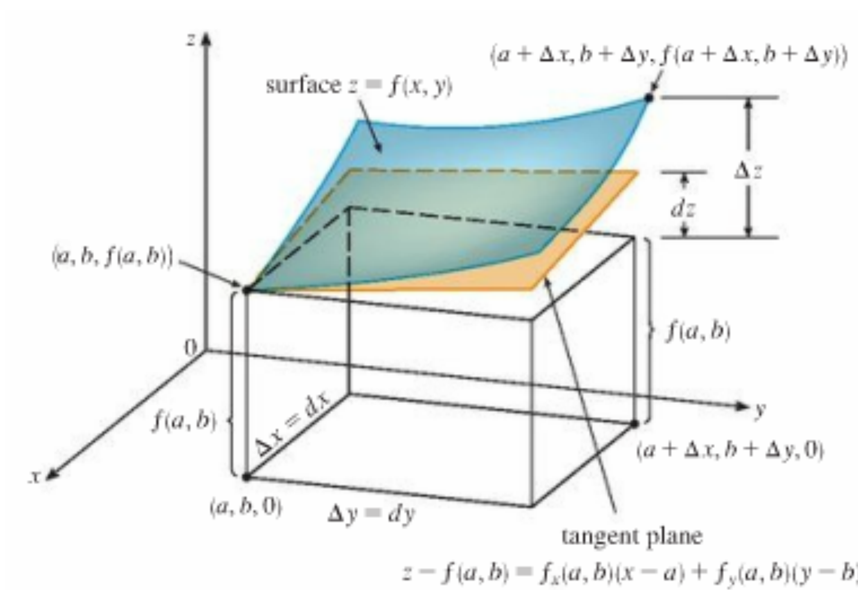
   (a) Find ∇h

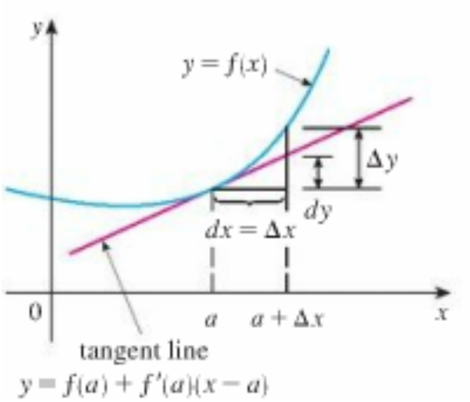


   (b) Find $D_{\overrightarrow{PQ}} f(1, 4)$
   where $P(\frac{1}{2}, 2)$ and $Q(0, 0)$.

   (c) What direction $\vec{u}$ would maximize the change in h when at $(1, 4)$?

Total Differential: df





Tangent Planes

Recall any of the following could be used to describe a line in $\mathbb{R}^2$:

$$y = mx + b \qquad ax + by = c$$

$$y - y_1 = m(x - x_1).$$

Consider an example of my favorite type of differential calculus question:

(a) Find the line tangent to the graph of $f(x) = \tan\left(\frac{\pi}{3}x\right)$ when x=1.

(b) Find the local linearization of f when $x = 1$.

(c) Use the linearization of f at $x = 1$ to approximate $f(1.1)$.

(d) How good is the approximation above? That is, what is the difference between your approximation above, and the actual value $f(1.1)$.

Recall any of the following could be used to describe a plane in $\mathbb{R}^3$:

$$\overrightarrow{n} \cdot (\langle x, y, z\rangle - \langle x_1, y_1, z_1\rangle) \qquad ax + by + cz = d$$

$$z - z_1 = m_x(x - x_1) + m_y(y - y_1).$$

(a) Find the plane tangent to the graph of $f(x, y) = \tan\left(\frac{\pi}{3}x\right) + \frac{1}{y}$ when $x = 1$ and $y = 1$.

(b) Find the local linearization of f when $x = 1$ and $y = 1$.

(c) Use the linearization of f at $x = 1$ and $y = 1$ to approximate $f(1.1, 1.1)$.

(d) How good is the approximation above? That is, what is the difference between your approximation above, and the actual value $f(1.1, 1.1)$.