

# Series

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Let the *sequence*  $a_n = \left(\frac{1}{2}\right)^n$  and the *series*  $s_n = \sum_{i=1}^n a_i$ .

(a) Write down the first 3 terms of  $a_n$ .

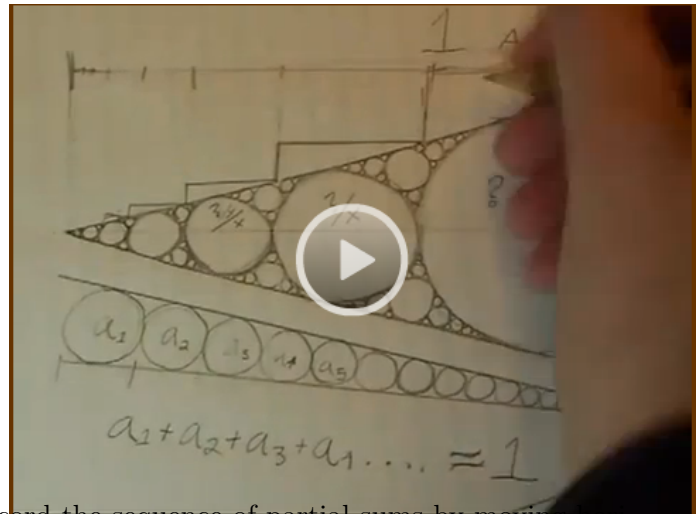
(b) Find  $\lim_{n \rightarrow \infty} a_n$

(c) Write down the first 3 terms of  $s_n$ .

(d) Hypothesize  $\lim_{n \rightarrow \infty} s_n$

Adapted to complement  
ViHart's *Infinity Elephants* video.

- (a) Consider the sequence  $a_n = \frac{1}{3}^n$ .
- (b) Write down a few of the terms in the sequence  $a_n$ .

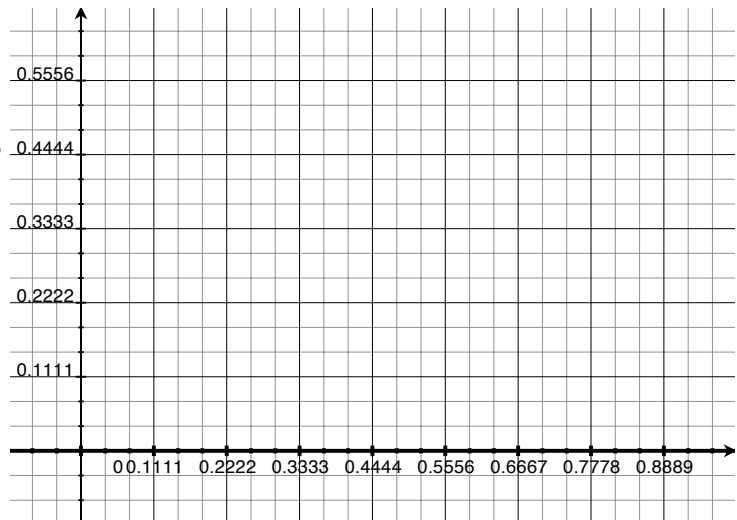


We will follow Vi Hart's lead and record the sequence of partial sums by moving horizontally across the page.

- (c) Draw a box (or an elephant) that is  $\frac{1}{3}$  units high &  $\frac{1}{3}$  units wide with the lower left "corner" on the origin.

The height will correspond with  $a_1$  and the total width across the page corresponds with  $s_1$ .

- (d) Draw the second box that has height  $a_2$  immediately to the right of the box (or elephant) you drew in (b) (that is with the lower left "corner" at the point  $(\frac{1}{3}, 0)$ ).



- (e) Identify the length that corresponds with  $s_2$ .
- (f) Draw a third box (or elephant) with height and width  $a_3$  immediately to the right of the box (or elephant) you drew in (c).
- (g) Identify the length that corresponds with  $s_3$ .
- (h) It gets difficult to repeat the procedure for  $s_4$ ,  $s_5$  and so on, but can you identify  $\lim_{n \rightarrow \infty} s_n$  is on the graph above?

- (i) Just as Vi Hart drew a line that skimmed across the tops of the circles, draw a line that skims the top right hand side of the boxes. Find the equation of the line you just drew.

- (j) Find  $\lim_{n \rightarrow \infty} s_n$