## Sequences \& Series

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Write out the first few terms of the sequence defined by $a_{n}=\left(1-.2^{n}\right)$.
2. Find a formula for the $n$th term $\left(b_{n}\right)$ in the sequence $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots\right\}$

A sequence $\left\{a_{n}\right\}$ has a limit $L$, denoted $\lim _{n \rightarrow \infty} a_{n}=L$ if we can make the terms $a_{n}$ as close to $L$ as we like by taking $n$ sufficiently large. If $\lim _{n \rightarrow \infty} a_{n}$ exists, the sequence converges, otherwise it diverges.
3. Revisit \#1 and 2 above and determine if the sequence converges or diverges. If the sequence converges, hypothesize what the limit is.
4. Recall in \#2 you found a formula for $b_{n}$.
Plot $\left(n, b_{n}\right)$ for a few of the terms in the sequence to confirm your work in (3).

|  |  |  |  | $y_{4}^{5}$ |  |  |  |  |  |
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5. Let $\alpha_{1}=1$ (an initial condition) and recursively define the sequence $\alpha_{n}=\frac{\alpha_{n-1}^{2}}{2}$.
(a) Write down the first few terms of the sequence.
(b) Determine if the sequence converges or diverges. Yes, this one is harder than (1), just try to make headway.
6. Let the sequence $\beta_{n}=\left(\frac{1}{2}\right)^{n}$ and the series $s_{n}=\sum_{i=1}^{n} \beta_{i}$.
(a) Write down the first 3 terms of $\beta_{n}$.
(b) Find $\lim _{n \rightarrow \infty} \beta_{n}$
(c) Write down the first 3 terms of $s_{n}$.
(d) Hypothesize $\lim _{n \rightarrow \infty} s_{n}$
