Exam 2 **TMath 126** Practice

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

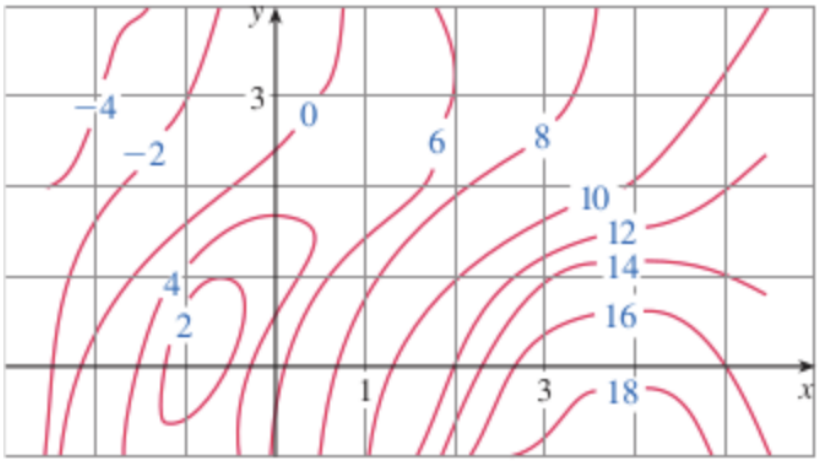
   Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) Let f be a function of x and y. If $\nabla f(c,d) = (2,1)$, then the vector $\langle 2,1\rangle$ is tangent to the contour line of the surface of f at $(c,d,f(c,d))$.

   (b) $\int_{-1}^{2}\int_{0}^{6} x^2\sin(x-y)\,dx\,dy = \int_{0}^{6}\int_{-1}^{2} x^2\sin(x-y)\,dy\,dx$

   (c) $\int_{-1}^{x}\int_{0}^{6} x^2\sin(x-y)\,dx\,dy = \int_{0}^{6}\int_{-1}^{x} x^2\sin(x-y)\,dy\,dx$

2. Let f have the contour lines shown on the right.



(a) Estimate $f(2,1)$

(b) Sketch the direction of the vector $\nabla f(2,1)$ on the graph.

(c) Identify one critical point on the graph of f and identify it as a local minimum, maximum or neither.

(d) Let $\vec{u} = \langle 3, -1 \rangle$ Determine whether the directional derivative of f at point $(-1, 3)$ along $\vec{u}$ is positive, negative, or zero.
Justify your answer.

(e) Estimate the volume bounded by f above the rectangle $3 \leq x \leq 5$ and $0 \leq y \leq 3$. Be clear about what choices you are making to estimate the volume.

3. You are given the following data of a function $g(x, y)$. Your boss wants you to approximate $g(.8, 1.4)$ and wants to be convinced you're doing something sophisticated. Find a linear approximation for your boss and explain your choices (there are many that you will make!).

x	y	$g(x, y)$
0.55	1.2	27
0.65	1.0	31
0.65	1.1	29
0.75	1.2	50

4. Consider the double integral

$$\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos(x)\sqrt{1+\cos^2 x}\, dxdy$$

(a) Sketch the region in the xy-plane where the integral is taken over.

(b) Switch the order of integration.

(c) Compute the double integral.

For the following problem you will outline (not actually find!) a solution. Make sure your outline includes:

(a) definitions of variables used,

(b) identifying the function that needs to be optimized,

(c) boxing systems of equations that need to be solved (but do not solve them!), &

(d) explaining how you would verify your work is correct (ie a maximum)

5. Find the maximum and minimum volumes of a rectangular box with the constraints that the surface area is 1500cm^2 and total edge length is 200cm.

6. Common blood types are determined by three alleles, A, B, and O. If p is the percent of allele A in the population, q is the percent of allele B in the population and r is the percent of allele O in the population then the proportion of individuals with a mixed blood type (e.g. AB, AO or BO) is $P(p, q, r) = 2pq + 2pr + 2qr$. Find the maximal P value.