Exam 2

TMath 126

Practice

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

Let \overrightarrow{a} , \overrightarrow{b} , and \overrightarrow{c} be vectors in \mathbb{R}^3 . Recall that \cdot refers to the dot product, and \times refers to the cross product.

(a) Let f be a function of x and y. If $\nabla f(c, d) = (2, 1)$, then the vector $\langle 2, 1 \rangle$ is tangent to the contour line of the surface of f at (c, d, f(c, d)).

(b) If $f_x(2,3) = 0$ then (2,3, f(2,3)) will be a critical point.

(c) To optimize the function $f(x, y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$ we would need to solve the following system of equations:

$$\left\{\begin{array}{c}
ye^{xy} = 3x^2 \\
xe^{xy} = 3y^2 \\
x^3 + y^3 = 16
\end{array}\right\}$$
(1)

- 2. Let f have the contour lines shown on the right.
 - (a) Estimate f(2,1)
 - (b) Sketch the direction of the vector $\nabla f(2, 1)$ on the graph.



- (c) Identify one critical point on the graph of f and identify it as a local minimum, maximum or neither.
- (d) Let $\overrightarrow{u} = \langle 3, -1 \rangle$ Determine whether the directional derivative of f at point (-1, 3) along \overrightarrow{u} is positive, negative, or zero. Justify your answer.

(e) Estimate a point (a, b) so that $f_x(a, b) > 0$ and $f_y(a, b) > 0$.

3. You are given the following data of a function g(x, y). Your boss wants you to approximate g(.8, 1.4) and wants to be convinced you're doing something sophisticated. Find a linear approximation for your boss and explain your choices (there are many that you will make!).

x	y	g(x,y)					
0.55	1.2	27					
0.65	1.0	31					
0.65	1.1	29					
0.75	1.2	50					

 4. Let f(x, y) = x² sin(x) - sin(y). (a) Draw sections of the contour map/the level curves of f when z = 1 and z = 2. Label the curves! 					y ₄ 3 2 1					
	-4	-3	-2	-1	0 -1 -2 -3 -4	1	2	3	4	x ⁵

(b) [3] Find $f_x(x, y)$

(c) Find $D_{\overrightarrow{u}}(2,0)$ where $\overrightarrow{u} = \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$.

For the following problem you will outline (not actually find!) a solution. Make sure your outline includes:

- (a) definitions of variables used,
- (b) identifying the function that needs to be optimized,
- (c) boxing systems of equations that need to be solved (but do not solve them!), &
- (d) explaining how you would verify your work is correct (ie a maximum)
- 5. Find the maximum and minimum volumes of a rectangular box with the constraints that the surface area is 1500cm^2 and total edge length is 200cm.

6. Common blood types are determined by three alleles, A, B, and O. If p is the percent of allele A in the population, q is the percent of allele B in the population and r is the percent of allele O in the population then the proportion of individuals with a mixed blood type (e.g. AB, AO or BO) is P(p,q,r) = 2pq + 2pr + 2qr. Find the maximal P value.