Exam 1 # TMath 126 Practice

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) If $\vec{u} \cdot \vec{v} = 0$, then $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

   (b) $(\vec{u} \times \vec{w}) \cdot \vec{w} = 0$

   (c) $\dfrac{\vec{u} \cdot \vec{v}}{||\vec{u}||||\vec{v}||} = \dfrac{\vec{u}}{||\vec{u}||} \cdot \dfrac{\vec{v}}{||\vec{v}||}$.

   (d) The line $(2 + 3t, -4t, 5 + t)$ where $t \in \mathbb{R}$ intersects the plane $4x + 5y - 2z = 18$ at the point $(-4, 8, 3)$.

   (e) If $\vec{r}(t) = \langle t^2, \ln(et), t^3 - 3t \rangle$, then the line tangent to $\vec{r}(1)$ is:

   $$\langle x, y, z \rangle = \langle 1, 1, -2 \rangle + \langle 2t, \frac{e}{t}, 3t^2 - 3 \rangle$$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

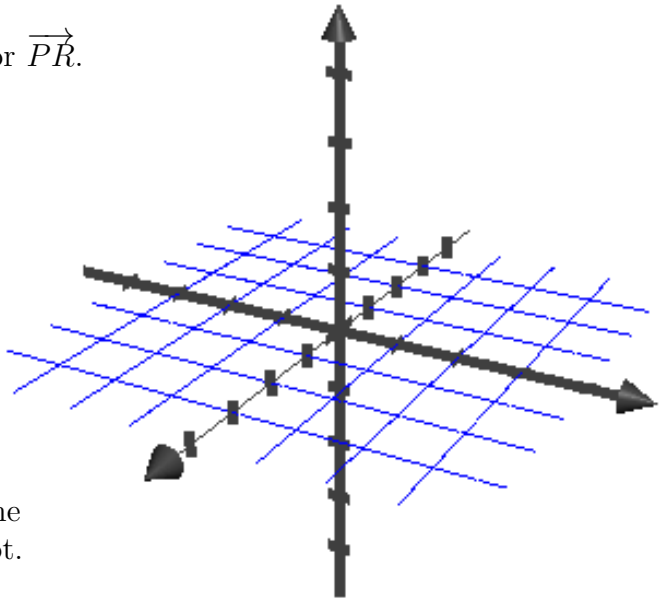
2. Consider the points: $P(1, 3, 2)$, $Q(3, -1, 6)$, and $R(5, 2, 0)$. Also let $S(3, 6, 1.5)$ and $T(-9, -14, -12.5)$.

   (a) Plot the points P, Q, and R.

   (b) Find the components of the vector $\overrightarrow{PR}$.

   (c) Find the length of $\overrightarrow{PR}$.

   (d) Draw the vector $\overrightarrow{PR} - 2\overrightarrow{j}$ and then write its components.



   (e) Use calculus methods to determine if $\triangle PQR$ is a right triangle or not.

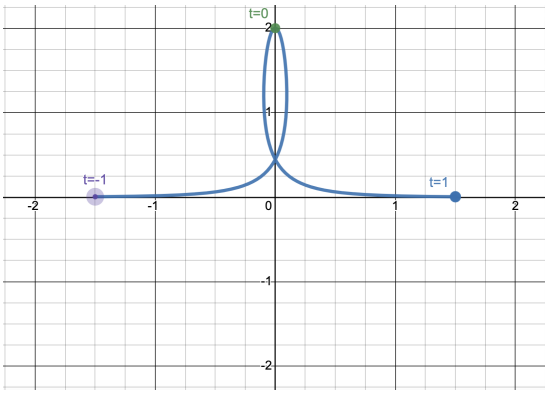
   (f) Find the equation of the plane that passes through P, R, and Q.

   (g) Does the line that passes through S and T intersect the plane you found in part (a)? Justify yourself.

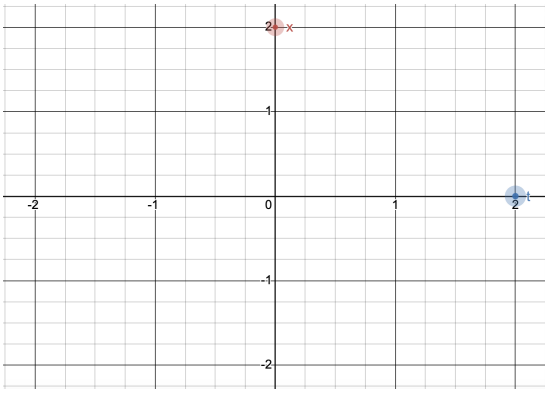
3. A tow truck drags a stalled car along a road. The chain makes an angle of 30° with the road and tension in the chain is 1600 Newtons. How much work (in J) is done by the truck in pulling the car 1km?

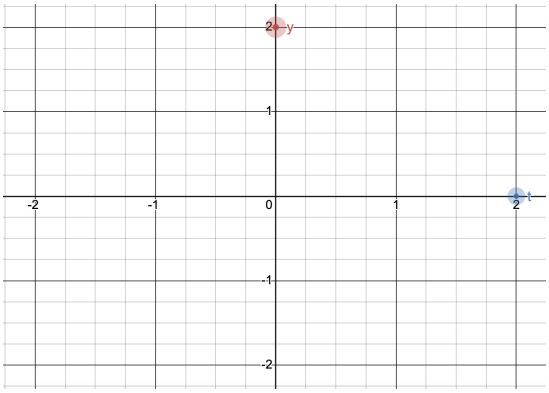
4. Consider the parametric curve $x = f(t)$, $y = g(t)$ where $-1 \leq t \leq 1$, graphed below for the following questions.

   (a) Looking at the graph, approximate where $\frac{dy}{dx}$ is not defined. (Report either a point on the graph or an approximate t value.)



   (b) Sketch the equations $x = f(t)$ and $y = g(t)$ on the pair of axis below.

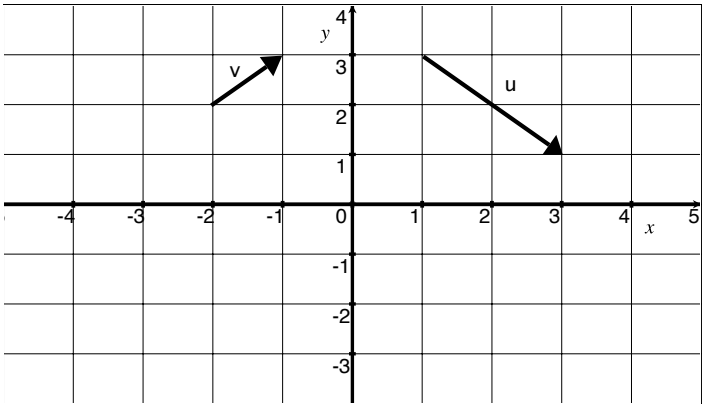




   (c) Given the following information, find the (approximate) line tangent to the curve $x = f(t)$, $y = g(t)$ when $t = \frac{1}{2}$. Use whatever form of a line you like (eg. parametric, slope-intercept, standard, etc)

$f\left(\frac{1}{2}\right) \approx 0$ $\quad$ $g\left(\frac{1}{2}\right) \approx .45$ $\quad$ $f'\left(\frac{1}{2}\right) \approx 1$ $\quad$ $g'\left(\frac{1}{2}\right) \approx -2.68$

5. Consider the vector $\overrightarrow{v}$ and $\overrightarrow{u}$ shown to the right.



   (a) Draw the vector $-3\overrightarrow{v}$.

   (b) Draw the vector $2\overrightarrow{v} - \overrightarrow{u}$.

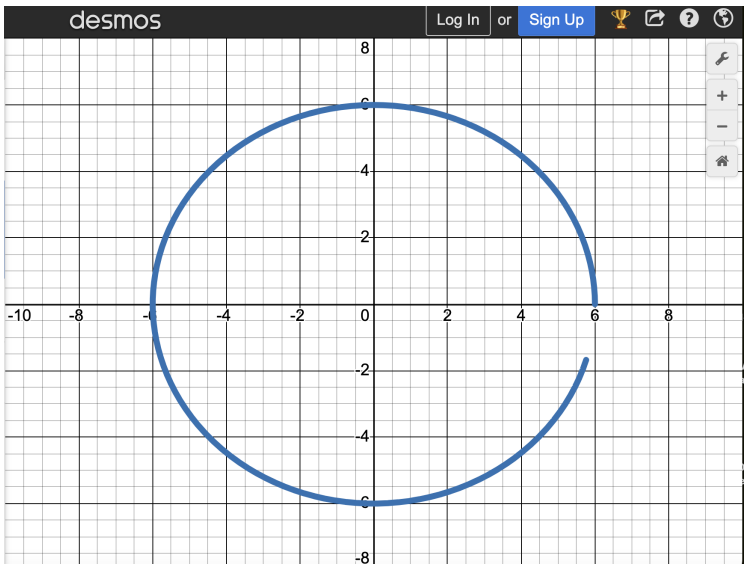
   (c) Find the projection of $\overrightarrow{u}$ onto $\overrightarrow{v}$.

6. We define $\overrightarrow{r}(t)$ by: $x(t) = 1 + t^4$, $y(t) = te^{-t}$, and $z(t) = \sin(2t)$.

   (a) Find the line tangent to the curve $\overrightarrow{r}(t)$ when $t = 0$.

   (b) Find the length of the arc traced by $\overrightarrow{r}(t)$ from $t = 0$ to $t = 5$.

7. A plane's position is traced by a parameterized curve: $x_p(t) = t^2 - 9$ and $y_p(t) = 2 - t$ (in km). Similarly, parameterized curves for a helicopter's position is $x_h(t) = 6\cos(t)$ and $y_h(t) = 6\sin(t)$ (in km). The helicopter's path is traced below for $t = 0$ to 10.



(a) As t increases, indicate the direction of the helicopter's path by adding an arrow to the path graphed.

(b) Sketch the path of the plane from $t = 0$ to $t = 8$.

(c) Set up the expression that will return the distance traveled by the helicopter between $(6, 0)$ and $(5.6568, -2)$. Make sure your answer can be completed with technology, you do *not* need to find the numeric answer!

(d) Find the coordinates of any points where the two paths intersect.

(e) Does the plane ever collide with the helicopter? Provide justification for your answer.